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ABSTRACT

This technical specialty course is one of thirty-five courses designed for use by two-year postsecondary institutions in five nuclear technician curriculum areas: (1) radiation protection technician, (2) nuclear instrumentation and control technician, (3) nuclear materials processing technician, (4) nuclear quality-assurance/quality-control technician, and (5) nuclear reactor (plant) operator trainee. This course, Reactor Physics, contains eight modules: Neutron Physics; Cross Sections and Reactions; Fission Rate and Reactor Power; Time Independent Nuclear Theory; Time Dependent Nuclear Theory; Reactivity; Fuel Moderator and Control Systems; Internal Effects on Reactivity; and Reactor Operating Characteristics. Each module follows a typical format including the following sections: introduction, module prerequisites, objectives, subject matter, exercises, materials, procedures, data tables, reference materials, and tests. (A master index and suggested program planning guide are also available--see CE 026 856 and CE 026 639-643.) (LRA)

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Nuclear Technology Series

COURSE 12
REACTOR PHYSICS

TECHNICAL EDUCATION RESEARCH CENTER - SW

4201 LAKE SHORE DRIVE - SUITE 111

WACO, TEXAS 76710

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P R E F A C E

ABOUT NUCLEAR TECHNOLOGY MODULES

The modules - developed by TERC-SW for use in two-year postsecondary technical institutions to prepare technicians for employment - are useful in industry for updating employees in company-sponsored training, and as laboratory texts in engineering schools. The principles, techniques and skills taught in the modules, based on tasks that nuclear technicians perform, were obtained from a nationwide advisory committee of employers of nuclear technicians. Each module was written by a working technical expert and tested with students.

A module contains the following elements:

Introduction, which identifies the topic and often includes a rationale for studying the material.

Module Prerequisites, which identify the material a student should be familiar with before studying the module.

Objectives, which clearly identify what the student is expected to know for satisfactory module completion. The objectives, stated in terms of action-oriented behaviors, include such action words as operate, measure, calculate, identify and define, rather than words with many interpretations, such as know, understand, learn and appreciate.

Subject Matter, which presents the background theory and techniques supportive to the objectives of the module. Subject matter is written with the technical student in mind.

Exercises, which provide practical problems to which the student can apply this new knowledge.

Materials, which identify the equipment required to complete the laboratory procedure.

Procedures, which is the experiment section, or "hands-on" portion of the module (including step-by-step instruction) designed to reinforce student learning.

Data Tables, which are included in most modules for the first year (or basic) courses to help the student learn how to collect and organize data.

Reference Materials, which are included as suggestions for supplementary reading/viewing for the student.

Test, which measures the student's achievement of the prestated objectives.

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The Nuclear Technology Series will consist of five curricula when completed:

- Nuclear Reactor (Plant) Operator Trainee.
- Radiation Protection Technician.
- Nuclear Materials Processing Technician.
- Nuclear Quality-Assurance/Quality-Control Technician.
- Nuclear Instrumentation and Control Technician.

The completed Nuclear Reactor (Plant) Operator Trainee curriculum consists of the following sixteen technical specialty courses made up of 114 instructional modules:

- Radiation Physics.
- Radiation Protection I.
- Principles of Process Instrumentation.
- Heat Transfer and Fluid Flow.
- Introduction to Nuclear Systems.
- Instrumentation and Control of Reactors and Plant Systems.
- Reactor Operations.
- Reactor Safety.
- Reactor Auxiliary Systems.
- Power Plant Systems.
- Radiation Detection and Measurement.
- Reactor Physics.
- Power Plant Chemistry.
- Introduction to Quality Assurance/Quality Control.
- Metallurgy and Metals Properties.
- Mechanical Component Characteristics and Specifications.

Availability: Instructional modules are available individually or by courses from:

Technical Education Research Center - Southwest
4201 Lake Shore Dr., Suite 111
Waco, Texas 76710
Telephone (817) 772-9756

Printed:

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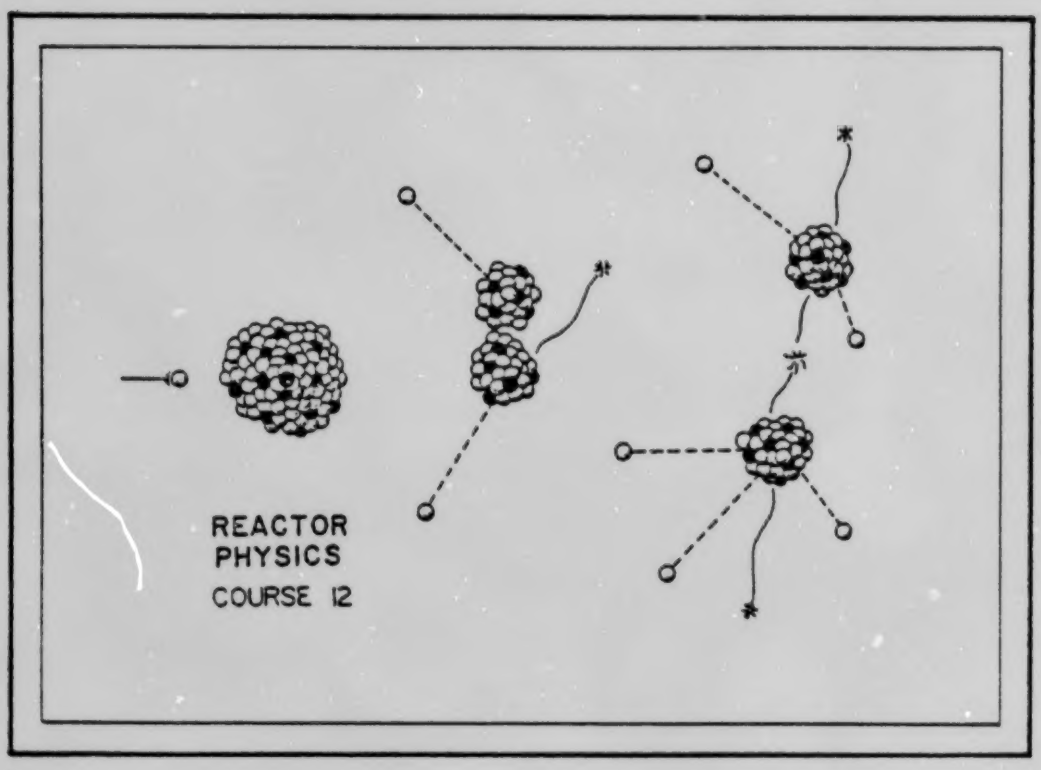
NUCLEAR TECHNOLOGY

MODULE 12-1 NEUTRON PHYSICS. -

RONALD L. KATHREN AND LARRY B. CHURCH



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TECHNICAL EDUCATION RESEARCH CENTER — SOUTHWEST
4201 LAKE SHORE DRIVE WACO, TEXAS

COURSE MODULES

COURSE 12 REACTOR PHYSICS

MODULE 12-1 Neutron Physics

MODULE 12-2 Cross Sections and Reactions

MODULE 12-3 Fission Rate and Reactor Power

MODULE 12-4 Time Independent Nuclear Theory

MODULE 12-5 Time Dependent Nuclear Theory

MODULE 12-6 Reactivity - Fuel Moderator and Control Systems

MODULE 12-7 Internal Effects on Reactivity

MODULE 12-8 Reactor Operating Characteristics

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INTRODUCTION

This course is designed to expose the student to fundamentals of nuclear reactor operation, including the fission process, which is caused by a neutron striking a fissionable atom. This first module discusses the neutron and its measurement. The neutral neutron does not directly cause ionization and must be measured by some other means.

MODULE PREREQUISITES

The student should have completed a basic course in elementary atomic and nuclear structure, such as Course 1, "Radiation Physics," and a first-year course in chemistry and/or physics. A general knowledge of radiation detection and reactors is also necessary; therefore Course 11, "Radiation Detection and Measurement," or Course 5, "Introduction to Nuclear Systems," should have been completed.

OBJECTIVES

Upon completion of this module, the student should be able to:

1. Discuss general characteristics of neutrons as presented in this module, including charge, mass, energy, range and penetrating ability.
2. List three mechanisms (nuclear reactions) by which radioactive sources can be used to release neutrons.
3. Write a nuclear equation for at least one type of radioactive neutron source.
4. Calculate neutron energy at thermal equilibrium and explain why thermal neutrons are so denoted.
5. Calculate neutron energy (given velocity or converse).
6. Calculate flux from a point source (given source strength and distance from source).
7. List and describe four categories of neutrons, based on energy.
8. List and discuss four techniques of neutron detection, giving physical principles and present applicability.
9. Define:
   - Flux.
   - Fluence.
   - Thermal neutrons.
   - Free neutrons.
   - D-T reaction.
   - D-D reaction.
   - Proton recoils.
10. Use a survey meter to detect neutrons.

The neutron is a subatomic particle. It has no charge, and its at-rest mass is approximately 1 atomic mass unit (amu) (the precise at-rest mass of the neutron is 1.008665 amu, greater by only 0.001388 amu than that of the proton). Neutron radius is about 8×10^{-14} cm, approximately equal to that of the proton.

Since the at-rest mass of the electron is only 0.000549 amu, unless an additional 0.000839 amu (0.7811 MeV) is somehow provided (perhaps a binding energy), it is obvious that a neutron is not simply a combination of a proton with an electron (this would violate the law of convervation of energy).

After discovery of the neutron, it was found that neutrons and protons (within a nucleus) interact constantly by transfer of other charged and neutral particles, called mesons (very light mass subatomic particles), between neutron-proton, proton-proton or neutron-neutron pairs. These so-called nucleon exchange forces are necessary to hold the nucleus together. For a variety of complex reasons, the neutron should be considered a fundamental atomic particle along with the proton and electron.

FREE NEUTRONS

Free neutrons (i.e., neutrons that exist outside a nucleus) can be produced in cosmic ray interactions. When a heavy, high-energy cosmic-ray particle (such as a proton

with a kinetic energy of several hundred MeV) collides with an atomic nucleus in the upper atmosphere, the nucleus may break up into smaller units. These smaller units may be free neutrons, or protons and other nuclear fragments. Free neutrons also occur in nature, resulting from spontaneous fission of heavy elements, such as ^{244}Pu, which is found in the earth's crust.

Sometimes, a free neutron behaves like a radioactive nuclide, undergoing beta decay to become a proton according to the reaction:

$$^{1}_{0}n \rightarrow {}^{1}_{1}p + {}^{0}_{-1}\beta + 0.782 \text{ MeV.} \quad \text{Equation 12-1-1 (Equation = Eq.)}$$

While the half-life of this reaction is 10.6 minutes, free neutrons are rapidly captured by nuclei - virtually all free neutrons are captured within a fraction of a second.

NEUTRON SOURCES

Since the number of free neutrons is very small, neutrons in quantity must be obtained from other sources for use in reactors, research laboratories or other applications. Large numbers of neutrons can be produced by appropriate nuclear reactions or from the spontaneous fission of very heavy elements. Neutron sources can be divided into two

broad categories:

Radionuclide sources.

Machine sources (e.g., accelerators, reactors).

RADIONUCLIDE SOURCES

Radionuclide sources, which are radioactive, are used to produce neutrons by one of three principal processes:

(α,n) reaction.

(γ,n) reaction.

Spontaneous fission (SF).

Radioactive neutron sources are the original source of neutrons, for it was bombardment of a light nucleus (beryllium) with alpha particles that led to the discovery of the neutron:

$$^{4}_{2}He + ^{9}_{4}Be \rightarrow ^{12}_{6}C + ^{1}_{0}n + Q \text{ (MeV)}. \qquad \text{Eq. 12-1-2}$$

This (α,n) reaction is one of two commonly used to produce neutrons. In the (α,n) reaction, an alpha emitter (such as radium or plutonium) is mixed with a low-Z (atomic number) element (such as beryllium or boron). The resulting (α,n) reaction (an α in, an n out) produces the energetic neutrons of energy Q (MeV).

Neutron yield depends on many factors, including:

Energy of the incident alpha particles.

Low-Z material being irradiated.

Completeness of mixture of the two ingredients.

Self-absorption of alpha particles and neutrons.
Source geometry.
Impurity content.

The other principal nuclear reaction commonly used to produce neutrons is the (γ,n), or photoneutron reaction. In this reaction, photoneutrons are produced by mixing nuclide-emitting, suitably high-energy gamma rays with a low-Z element (e.g., beryllijm or deuterium).

Only beryllium or deuterium can produce neutrons with gamma-ray energies below 5 MeV. For beryllium, the photoneutron production threshold is 1.67 MeV (for deuterium it is 2.23 MeV); thus, a source must produce gamma rays with energies above 1.7 MeV if beryllium is used.

Spontaneous fission (SF) sources are heavy elements, generally all artificially produced, that undergo fission as a mode of radioactive decay. The fission is accompanied by emission of two or three neutrons and several gamma rays. SF sources are radioactive, in the common sense, and also undergo alpha decay; thus, two competitive decay modes, each has its own characteristic half-life. The alpha decay mode has a greater probability of decay, hence a shorter half-life (Table 12-1-1).

MACHINE SOURCES

Machine sources of neutrons include reactors and particle accelerators. In general, these sources are not widely employed because of their sizes. A possible exception is the so-called D-T (deuterium-tritium) generator, a small positive-ion accelerator that utilizes the D-T reaction for generating neutrons by bombarding a tritium-rich target

TABLE 12-1-1

COMMON NEUTRON SOURCES AND THEIR PROPERTIES

SOURCE	REACTION	HALF-LIFE	MEAN NEUTRON ENERGY (MeV)	APPROX. YIELD (n/sec·Ci)	COMMENTS
^{124}SbBe	(γ,n)	60 days	0.024	1.6×10^6	Short half-life renders this source unsuitable for many applications; has high-intensity gamma field.
^{210}PoB	(α,n)	138 days	4.3	6.0×10^3	
^{210}PoBe	(α,n)	138 days	4.5	2.5×10^6	
^{210}PoLi	(α,n)	138 days	0.5	6.1×10^5	Yield relatively low.
^{226}RaBe	(α,n) (γ,n)	1640 years 1640 years	3.9 0.3	1.5×10^7 1.2×10^4	High intensity gamma field.
^{227}AcBe	(α,n)	21.8 years	4.6	$\sim 1.2 \times 10^6$	Not generally available.
^{228}ThBe	(α,n)	1.9 years	0.8	$\sim 2.5 \times 10^6$	Not generally available; high intensity gamma field.
^{238}PuBe	(α,n)	87.8 years	4.5	1.8×10^6	Common; used for calibration of instruments.
^{239}PuBe	(α,n) (SF)*	24,360 years 24,360 years	4.5 1.2¶	1.5×10^6 0.5	Common; used for calibration of instruments.
^{239}PuF$_4$	(α,n)	24,360 years	0.7	$\sim 10^6$	Physically relatively large, much self-absorption.
^{241}AmBe	(α,n) (SF)*	458 years	4.4¶ 1.2¶	2.2×10^6 1.9	Becoming common; good reactor starter source.
^{240}Pu	SF*	6600 years	1.2¶	3500*	Relatively weak for practical use.
^{244}Cm	SF*	18.4 years	1.2¶	1.5×10^5*	High gamma field.
^{252}Cf	SF*	2.65 years	1.2¶	4.3×10^9*	Coming into wide use as calibration and starter source.
^{249}Bk	SF*			3.6×10^5*	Not yet available in quantity.

* SF = spontaneous fission.
¶ Mean energy of fission neutron spectrum.
* Yield is given in n/sec-g.

($^3_1$H) with positive deuterium ions ($^2_1$H):

$$^2_1\text{H} + {}^3_1\text{H} \rightarrow {}^4_2\text{He} + {}^1_0\text{n} + Q \text{ (MeV)}.$$

Eq. 12-1-3

Q = 17.6 MeV; all generated neutrons have the same energy, ~ 14 MeV; the other 3.6 MeV is associated with the liberated α particle.

Another neutron-generating machine sometimes used is a D-D (deuterium-deuterium) generator; it utilizes a D-D reaction. This machine is analogous to the D-T generator except that the target is deuterium rich rather than tritium rich. Some commercial units are made with interchangeable targets so that neutrons can be generated by either a D-D or a D-T reaction.

Accelerators of various kinds - betatrons, cyclotrons, linacs, etc., - can be used to produce neutrons from a wide variety of nuclear reactions. In addition to the D-D and D-T reactions, the (p,n) reaction is also widely used in accelerators. In this reaction, a proton is accelerated into a target nucleus to produce neutrons:

$$ {}_{1}^{1}H + {}_{Z}^{A}X \rightarrow {}_{(Z+1)}^{A}Y + {}_{0}^{1}n + Q. \qquad \text{Eq. 12-1-4} $$

where

X = target nucleus.

Y = product after reaction.

[In general, fission-produced neutrons (from reactors) are used for research, educational and analytical purposes as well as for power production; the remaining modules in this course provide a more comprehensive consideration of this area.]

Neutron sources can produce neutrons over a wide range of energies (Figure 12-1-1). As can be seen (Table 12-1-1),

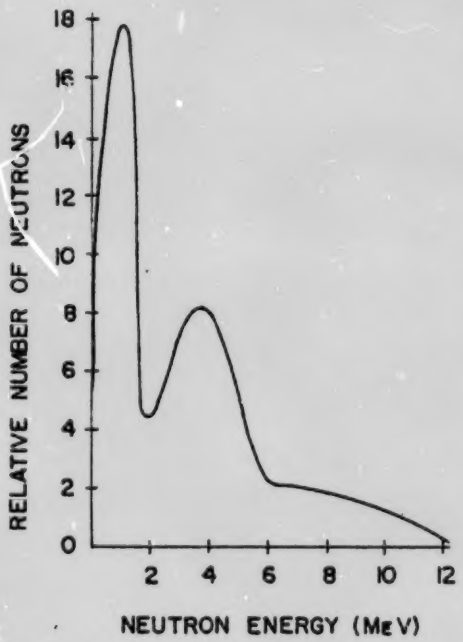

Figure 12-1-1.
Neutron energy spectrum from Ra-Be (α,n) source.

Ra-Be is primarily an (α,n) source. The spectrum shows peaks at about 1 MeV and 4 MeV, and a continuous (i.e., neutrons of all energies) spectrum from 0 to 12 MeV.

Most (α,n) reactions produce a neutron energy spectrum with a peak in the region of 3 to 5 MeV. Photoneutron sources may also produce continuous-energy spectra, or they may produce neutrons with only one or a few characteristic energies. The relatively common ^{124}Sb-Be source, usually made by surrounding a rod of ^{124}Sb with a sleeve or cladding of beryllium, emits neutrons in one single energy group: approximately 0.03 MeV.

Other neutron-producing nuclear reactions, such as the D-T, result in monoenergetic (single-energy) neutrons,

or a very narrow spectrum of energies. As was mentioned above, the D-T reaction produces neutrons with energies of 14 MeV, while the D-D reaction produces neutrons with energies primarily in the few-MeV region.

A fission neutron spectrum (Figure 12-1-2) is signifi-

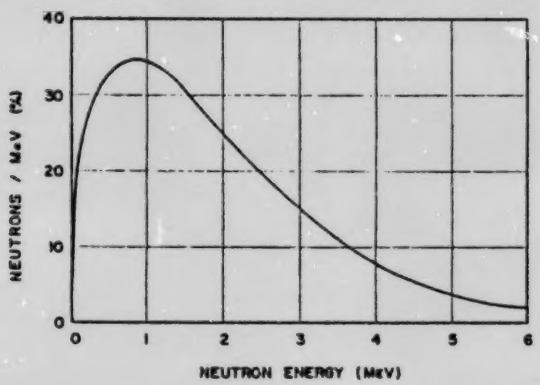

Figure 12-1-2. Energy distribution of neutrons from thermal fission of ^{235}U.

cantly different from spectra produced by other nuclear reactions (Figure 12-1-1). Fission neutrons are produced over a broad range of energies, from 0 to about 18 MeV. The number of neutrons in a given energy group is roughly inversely proportional to the log of the energy over the range from about 1 to 18 MeV. In this range, probability of fission goes up as incident neutron goes down, because each neutron has a greater chance of interaction with a nucleus if it is moving slowly. (It spends more time in the vicinity of a target nucleus!) A peak in neutron production rate occurs at about 0.7 MeV. The decrease below

0.7 MeV is due to self-absorption in the source.

It is instructive to characterize neutron spectrum in terms of its average neutron energy:

$$\bar{E} = \frac{\Sigma_o^i n_i E_i}{N} \qquad \text{Eq. 12-1-5}$$

where

$\bar{E}$ = average neutron energy.

i = any specific group of neutrons with energy E_i, or between E_1 and E_2 centered about E_i.

n_i = number of neutrons with energy E_i, or with energies between E_1 and E_2 centered about E_i.

N = total number of neutrons.

Σ_o^i = sum of products of N_i and E_i over all energy groups (from group 0 to group i).

CLASSIFICATION OF NEUTRONS

Neutrons can be characterized in many different ways. One is by their method of production as with the neutron sources. More commonly, however, neutrons are categorized according to their energy; the type of nuclear reaction or interaction with matter is strongly dependent upon neutron energy.

Neutron energies generally range from about 10^{-4} eV to about 1000 MeV - more than 13 orders of magnitude.

Commonly, neutrons are put into four broad energy categories:

Thermal.
Intermediate.
Fast.
Relativistic.

Thermal neutrons are neutrons in thermal equilibrium with their surroundings. This statement means that neutron energy is about the same as the energy of surrounding atoms that are in constant vibrational motion. This motion depends on temperature, hence the term "thermal neutrons."

Actually, a whole distribution of neutron energies exists in this situation (Figure 12-1-3) - known as the

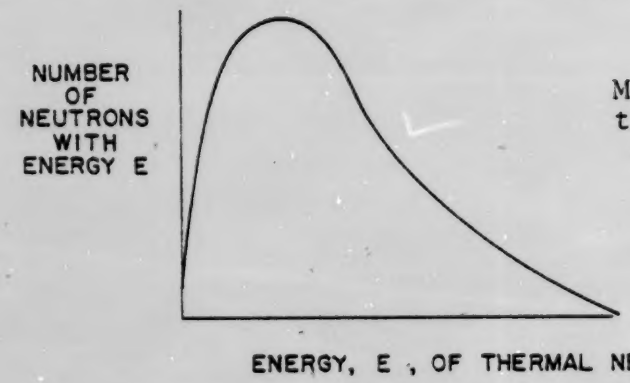

Figure 12-1-3. Maxwell-Boltzmann distribution of neutrons in thermal equilibrium. Peak occurs at 0.025 eV for room temperature.

Maxwellian, or Maxwell-Boltzmann, distribution. The most probable energy of a thermal neutron can be calculated from kinetic theory:

$$E = KT \qquad \text{Eq. 12-1-6}$$

where

E = energy of neutron, ergs.
K = Boltzmann constant
= 1.38×10^{-16} erg/°K.
T = temperature in °K (absolute temperature).

EXAMPLE
At room temperature (68°F ~ 20°C ~ 293°K), the most probable energy is easily calculated as follows:

$E = 1.38 \times 10^{-16}$ ergs/°K x 293°K
$= 4.04 \times 10^{-14}$ ergs

or, because there are 6.24×10^{11} eV/erg,
$E = 6.24 \times 10^{11}$ eV/erg x 4.04×10^{-14} ergs
= 0.025 eV at 20°C.

Thermal neutrons are generally considered to be monoenergetic at this most probable energy, and this simplifying assumption provides sufficient accuracy for most pur-

poses. Thermal neutrons, for practical purposes, are taken to be neutrons with an energy of 0.025 eV.

Because neutron energy is actually kinetic energy, it is possible to relate the velocity of a neutron to its energy by the following relationship from classical physics:

$$KE = 1/2\ mv^2 \qquad \text{Eq. 12-1-7}$$

where

KE = kinetic energy (energy of motion) of neutron, ergs.
m = neutron mass, grams.
v = neutron velocity, cm/s.

Thus, one erg is a gram cm^2/s^2.

For a 20°C thermal neutron, KE = 0.025 eV = 4.04×10^{-14} erg.

EXAMPLE

Velocity of a thermal neutron may be calculated as follows:

4.04×10^{-14} ergs
$= 1/2 \times 1.67 \times 10^{-24}$ grams $\times\ v^2$
$v^2 = 4.84 \times 10^{12}\ cm^2/s^2$ or ergs/grams
$v = 2.2 \times 10^6$ cm/s = 2200 m/s

Thus, the velocity of a thermal neutron, in equi-

librium at 20°C (room temperature), is 2200 m/s, or about 5000 miles per hour.

Even though thermal neutrons are taken, for practical purposes, to have energies of 0.025 eV, they actually range up to about 0.5 eV. The upper end of the thermal region (from 0.025 eV to 0.5 eV) is sometimes referred to as the epithermal region, and neutrons with energies in this range are called "epithermal neutrons."

"Intermediate neutrons" are those with energies in the region 0.5 eV to 10 keV.

"Fast neutrons" are those with energies between 10 keV and 1 MeV. From a radiation protection standpoint, these are the most hazardous neutrons. The velocity of fast neutrons can be calculated in the manner used for thermal neutrons.

"Relativistic neutrons" are those with energies greater than 10 MeV. This energy, which is generally greater than the binding energy of most nucleons, is particularly effective at fragmenting nuclei. The term relativistic comes from Einsteinian physics, which notes that, as the velocity of an object approaches the velocity of light, it gets heavier and heavier until, at the velocity of light, its mass becomes infinite.

The four energy groups are most useful in characterizing types of nuclear interactions that predominate in each energy region. Thermal (also known as slow) neutrons are most commonly involved in nuclear reactions that result in their capture by a nucleus, which includes emission of gamma ray, and most fission reactions.

The absorption or capture of intermediate energy neutrons is marked by a series of peaks or points of high affinity for neutrons of a particular energy. These are known as resonance peaks; hence intermediate neutrons, sometimes known as resonance neutrons, are target specific. This feature is of great utility in analysis of materials. Fast neutrons exhibit a variety of nuclear reactions, but capture reactions do not necessarily predominate, although under some conditions they may. Examples of other reactions are (n,α), (n,p) and (n,2n).

Very high-energy or relativistic neutrons have the ability to produce complex nuclear reactions, including "spallation" (a reaction in which a target nucleus is broken up into several fragments).

There are other neutron classifications. Often, it is convenient to use two broad categories: fast and slow (thermal). Neither has an exact or precise definition. Other, more specific, characterizations are discussed in subsequent modules in this Course.

NEUTRON FLUX AND FLUENCE

Neutrons are most commonly and universally measured in terms of neutron flux, more commonly referred to as <u>flux</u> (ϕ). Flux is the number of neutrons passing through a unit area (1 cm^2) in a unit of time (1 s). Thus, neutron flux, a rate of neutron flow across a boundary (e.g., detection aperture), stated as neutrons per square centimeter per second (n/cm^2-s or $n/cm^2/s$), is independent of energy.

It is, of course, possible to qualify flux by use of an appropriate descriptor, such as "thermal" or "fast." Thus, one can quite properly and precisely talk of thermal flux ϕth, the number of thermal neutrons (Nth) per cm^2 per second.

Neutron flux can be calculated from sources of known strength and geometry by various mathematical expressions (most radionuclide neutron sources and accelerator targets are sufficiently small to be considered as point sources):

$$\phi = \frac{S}{4\pi d^2} \qquad \text{Eq. 12-1-8}$$

where

S = source strength (or yield), neutrons/s.

d = distance from source.

(Note: This calculation ignores absorption and scattering.)

Neutron flux is sometimes represented by the letters NV:

$$\phi = NV.$$

The quantity NV is simply a product of the number of neutrons/unit volume in (n/cm^3) and their velocity in (cm/s), which dimensionally has the appropriate units of flux, viz

n/cm^2-s. The NV expression is often found in analysis and reactor physics work.

If flux (or average neutron flux) is multiplied by time of irradiation (or exposure), the total number of neutrons passing through a unit area (1 cm^2) is obtained. This quantity, known as fluence, is expressed in units of n/cm^2. (Fluence is symbolized by the Greek letter ψ or by NVt.) In the NVt expression, t refers to time, and the unit is again dimensionally n/cm^2. Note that the fluence rate is merely flux and has the appropriate units. Occasionally, fluence is referred to as the "time integral of flux" or flux time integral. This expression is correct, having been derived from calculus notation, but is cumbersome. Hence, the simple term fluence is preferred.

Neutrons can also be measured in other units. The familiar unit of absorbed dose - the rad - can be used to quantify neutron dose, and the biological dose from neutrons can be expressed in units of rem (Roentgen equivalent man).

NEUTRON DETECTION

Because neutrons are uncharged, they cannot be directly detected by ordinary electronic means; however, because of their ability to cause nuclear reactions of various types, neutrons can be detected and measured by several indirect methods, which include the following:

Nuclear emulsions.

Fission counters.

BF_3 proportional counters.
Activation analysis.

Generally, these neutron measuring techniques require some means of defining the spectrum to obtain absolute flux measurements. Otherwise, only relative values of flux in a fixed spectrum are obtained.

NUCLEAR EMULSIONS

Nuclear emulsions are essentially thick pieces of photographic material that contain a high percentage of hydrogen. As neutrons pass through the emulsion, they knock some of the hydrogen atoms loose and give them energy. These charged particles, known as proton recoils, move a small distance through the photographic emulsion. When the film is developed and examined under a microscope, a tiny track is seen that marks travel of the proton recoil (Figure 12-1-4).

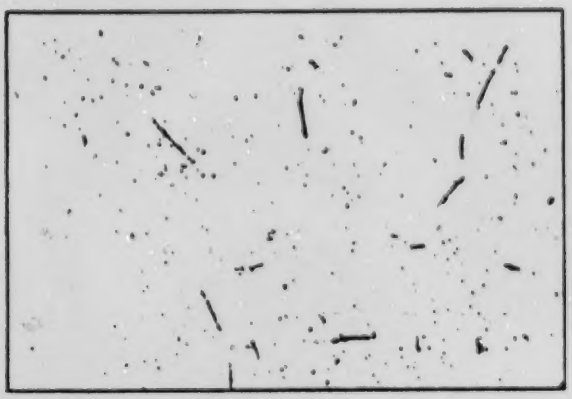

Figure 12-1-4.
Proton recoil tracks in nuclear emulsion.

For a given neutron spectrum, the number of tracks per unit area is proportional to the neutron fluence.

Nuclear emulsions are widely used as personnel monitoring devices.

(Note: It should be noted that nuclear emulsions are not very good for measuring neutron exposure because of variation of quality factor with energy, unless the spectrum is well characterized. Problems also arise because emulsions are insensitive to important parts of the neutron energy spectrum.)

FISSION COUNTERS

A fission counter is a device that utilizes uranium or some other fissionable material. When struck by neutrons of an appropriate energy, fission occurs and charged particles and gamma rays are produced. If the fissionable material has been used to line a counter tube (e.g., a Geiger-Mueller tube or proportional counter tube), a direct electronic readout of the fission events can be made, which provides a measurement of flux (the number of fissions per second is directly proportional to the neutron flux). A fission chamber provides a measurement of flux if the spectrum is known and the counter is properly calibrated.

BF_3 PROPORTIONAL COUNTERS

Perhaps the most widely used neutron flux detector is the BF_3 (boron trifluoride) proportional counter. This device works in the same fashion as a conventional proportional counter, except that, in this case, the proportional

counter tube is filled with BF_3 gas. When placed in a neutron flux, the following nuclear reaction occurs:

$$^{10}_{5}B + ^{1}_{0}n \rightarrow ^{4}_{2}He + ^{7}_{3}Li + Q.$$

Eq. 12-1-9

Alpha particles, produced inside the detector, are appropriately detected. A BF_3 proportional counter makes an excellent flux detector because it is especially sensitive to thermal neutrons.

ACTIVATION ANALYSIS

Activation analysis is a useful technique for monitoring fluence. Appropriate elements, placed in the neutron field, capture neutrons and become radioactive:

$$^{197}_{79}Au + ^{1}_{0}n \rightarrow ^{198}_{79}Au + Q.$$

Eq. 12-1-10

The radioactive ^{198}Au emits a beta particle with a half-life of 2.7 days. Thus, by determining the amount of radioactivity and making appropriate calculations, neutron fluence can be determined.

There are, of course, many other neutron detectors, including neutron scintillators, boron-lined chambers of various types, and a variety of β-γ detectors loaded with

materials that become activated. Also used are boron-loaded films and thermoluminescent materials, the latter being of particular value in personnel monitoring work.

NEUTRON SURVEY METERS

Virtually all neutron survey meters use BF_3-filled gaseous detector tubes and function as proportional counters. BF_3 tubes may be surrounded by paraffin, polyethylene or some other hydrogenous material, which acts to slow down and thermalize fast neutrons. This feature makes the instrument more sensitive because the $^{10}B(n,\alpha)\ ^{7}Li$ reaction is more likely to occur with thermal or slow neutrons than with fast neutrons.

Neutron survey meters can be used to gain information on neutron flux levels. Their readings must be carefully considered and evaluated because response of the instrument is highly dependent on neutron energy. Depending upon the amount of hydrogenous material surrounding the BF_3 tube, sensitivity can be several hundredfold greater for thermal neutrons that for fast neutrons; thus, in the same flux of a ^{226}Ra-Be source, which is mostly fast neutrons, and a fission source as moderated in a reactor, enormously different readings might be obtained. Neutron instruments should be appropriately calibrated, therefore, by using the same type of neutron spectrum as is to be measured. In particular, this calibration is important if flux readings are to be converted to dose rate (errors of several hundredfold are possible and could have serious consequences).

A technique sometimes used to crudely estimate thermal and fast flux is to compare the reading of a bare BF_3 proportional detector with one wrapped with cadmium. The cadmium absorbs virtually all the thermal neutrons, and, by noting the ratio of the bare-to-shielded readings, a crude idea of the spectrum can be obtained. Such techniques should be carried out by experienced individuals and should not supplant more accurate methods.

For personnel dose rate measurements, a BF_3 detector can be wrapped in a specially designed polyethylene-boron sleeve, which makes its response to neutrons approximate that from a human body. Such an instrument can be directly calibrated in units of rem/hr, or rad/hr.

EXERCISES

1. Use Table 12-1-1; calculate neutron flux expected at one foot (30.48 cm) from a 1.5 Ci ^{238}PuBe source.
2. Calculate the most probable energy of a thermal neutron in a reactor in which the temperature is 412°F; calculate its velocity.
3. Calculate velocity of a thermal neutron on a hot (110°F) summer day; on a cold (-10°F) winter day.
4. Write an equation for the (γ,n) reaction on beryllium.
5. Write a nuclear equation for the (D-D) reaction.

MATERIALS

1 neutron source, PuBe, 5 curies (80 grams).
1 neutron survey meter.
2 x 3 cycle log-log paper.

PROCEDURES

1. Use survey meter; measure flux or dose rate from a neutron source at these distances: 15 cm, 30 cm, 100 cm, 200 cm; plot flux versus distance on log-log paper; compare with calculated values based on source strength; discuss reasons for differences noted.
2. Repeat the above procedure; use a detector wrapped with cadmium. How do results compare with those above? Explain differences.

DATA TABLE

None.

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Available by writing the Superintendent of Documents, U.S. Government Printing Office, Washington, DC 20402.

1. Describe a typical neutron; use such characteristics as charge, mass, energy range and penetrating ability:

2. List three nuclear reactions by which radioactive sources can be made to emit neutrons:
   (1)
   (2)
   (3)

3. Write a nuclear equation associated with each of the following types of neutron sources:

   α,n.

   γ,n.

   Spontaneous fission.

4. Calculate energy of a thermal neutron at a temperature of 25°C:

5. Calculate kinetic energy of a neutron with a velocity of 5×10^6 cm/s:

6. Calculate flux from a 5-curie ^{238}PuBe at a distance of 10 feet:

7. List four neutron energy categories; include energy range associated with each:
   (1)
   (2)

(3)

(4)

8. List four techniques of neutron detection; include principles of each:

(1)

(2)

(3)

(4)

9. Define:

Thermal neutron.

Free neutron.

Flux.

Fluence.

D-T reaction.

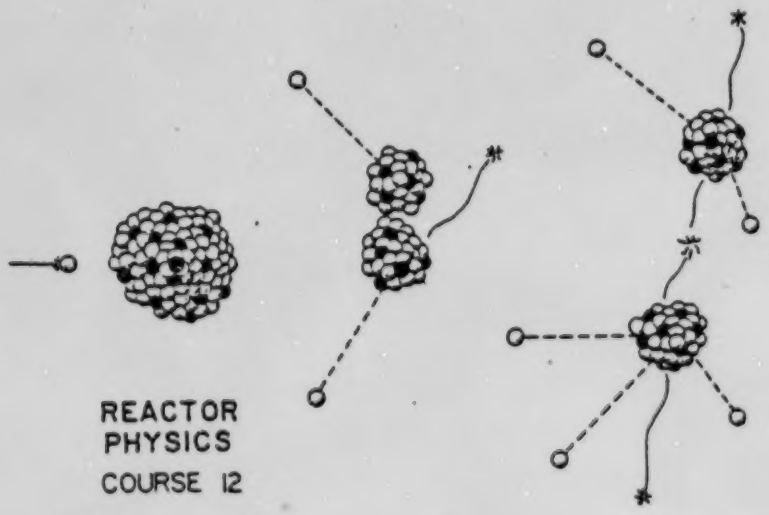
D-D reaction.

Proton recoils.

NUCLEAR TECHNOLOGY

MODULE 12-2 CROSS SECTIONS AND REACTIONS. -

RONALD L. KATHREN AND LARRY B. CHURCH



REACTOR
PHYSICS
COURSE 12

TERC-SW

TECHNICAL EDUCATION RESEARCH CENTER — SOUTHWEST
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COURSE MODULES

COURSE 12 REACTOR PHYSICS

MODULE 12-1 Neutron Physics

MODULE 12-2 Cross Sections and Reactions

MODULE 12-3 Fission Rate and Reactor Power

MODULE 12-4 Time Independent Nuclear Theory

MODULE 12-5 Time Dependent Nuclear Theory

MODULE 12-6 Reactivity - Fuel Moderator and Control Systems

MODULE 12-7 Internal Effects on Reactivity

MODULE 12-8 Reactor Operating Characteristics

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INTRODUCTION

This module describes several neutron interactions with matter. Possible interactions and their probabilities (as well as other concepts) need to be understood before the fission process is studied. "Cross section" gives an indication of the probability of a particular type of interaction taking place. Its application and methods of measurement are introduced, along with basic laboratory procedures demonstrating absorption, attenuation, thermalization and activation.

MODULE PREREQUISITES

The student should have completed Course 1, "Radiation Physics," a course (first-year level) in chemistry and/or physics, and Course 11, "Radiation Detection and Measurement," or Course 5, "Introduction to Nuclear Systems."

OBJECTIVES

Upon completion of this module, the student should be able to:

1. List three different types of reactions that produce

neutrons.

2. Write typical reaction equations for radioactive capture and charged-particle emission.
3. Describe processes that take place during these neutron scattering reactions:
   - Head-on collision with a small nucleus.
   - Glancing collision with a small nucleus.
   - Head-on collision with a large nucleus.
   - Glancing collision with a large nucleus.
4. Describe cross-section concept; state why it is not equal to cross-sectional area of a nucleus.
5. Use "Chart of the Nuclides" and Radiological Health Handbook to research scattering, absorption and fission cross sections.
6. List differences between microscopic and macroscopic cross sections.
7. Calculate macroscopic cross section, given only the radioisotope and its neutron density.
8. Draw a typical absorption cross section versus energy curve; explain significance of three major parts of curve.
9. Explain reason for reaction rate per cm^3 to be given as $\Sigma\phi$ and total reaction rate by $\Sigma\phi V$.
10. Define:
    - Nuclear reaction.
    - Elastic scattering.
    - Inelastic scattering.
    - Capture.
    - Compound nucleus.
    - Fission.
    - Cross section.
    - Microscopic cross section.

$\frac{1}{v}$.

Resonance.

Macroscopic cross section.

Albedo.

Cadmium ratio.

Neutron age.

Neutron reaction rate.

11. Experimentally determine macroscopic and microscopic absorption cross sections of a known material by using a portable neutron survey meter and a monoenergetic beam of neutrons.
12. Experimentally determine Cd ratio for a given neutron flux.

ELEMENTARY NUCLEAR REACTIONS

Nuclear reactions are those that occur within the nucleus of an atom. They are usually governed by internal forces, which can result in a radioactive nucleus, or by external forces, such as a bombardment by an energetic particle. Nuclear reactions may result in a variety of changes, including production of a nucleus of different elements or production of radiation.

ENERGETICS OF NUCLEAR REACTIONS

The general equation characterizing a nuclear reaction can be written as:

$$A + B \rightarrow C + D + Q. \qquad \text{Equation 12-2-1 (Equation = Eq.)}$$

In the equation, A, B, C and D are various particles (or nuclei), and Q refers to the necessary mass balance or energy term. An example of a nuclear reaction, taken from Module 12-1, "Neutron Physics," is the production of neutrons by alpha bombardment of beryllium nuclei:

$$^{4}_{2}He + ^{9}_{4}Be \rightarrow ^{1}_{0}n + ^{12}_{6}C + Q. \qquad \text{Eq. 12-2-2}$$

In this equation, not only is the reaction shown, but certain notations have been added to aid in illustrating basic rules of such reactions. These notations are atomic number, Z, and mass (or nucleon) number, A. The number of nucleons (protons plus neutrons) on the left side of the equation (4 + 9) exactly equals the number on the right side (1 + 12). Similarly, the atomic number [i.e., the number of nuclear charges on the left side (2 + 4) equals the number on the right side (0 + 6)]. Two of the three conservation rules governing nuclear reactions are illustrated:

In any nuclear reaction, the mass (nucleon) number is conserved.

In any nuclear reaction, the atomic number is conserved.

The third conservation rule:

In any nuclear reaction, energy is conserved.

This rule is incorporated in the Q term. If each reactant in the equation is carefully weighed, masses of those on the left side do not equal those on the right. The slight difference is shown by detailed calculation:

Rest mass of a $^{4}_{2}He$ atom	=	4.002603 amu
Rest mass of a $^{9}_{4}Be$ atom	=	9.012186 amu
Total mass, left side	=	13.014789 amu
Rest mass of a $^{1}_{0}n$ neutron	=	1.008665 amu
Rest mass of a $^{12}_{6}C$ atom	=	12.000000 amu
Total mass, right side	=	13.008665 amu

Mass difference = 13.014789 - 13.008665
= 0.006124 amu.

The mass difference (the value of Q in the equation) is known as "mass defect."

Because matter can neither be created nor destroyed, the missing mass must be accounted for. If $E = mc^2$ is used for accounting, the missing mass has been converted to energy and is actually the kinetic energy radiated or imparted to the products on the right side. The kinetic energy imparted to each particle is in inverse proportion to the mass of the products; thus, each neutron gets about 12 times as much of the kinetic energy (about 12/13 of the total) as does each carbon nucleus.

The apparent conversion of mass into energy in a nuclear reaction is explainable in terms of the binding energy needed to hold various nuclei together. In general, the amount of energy required to hold a large nucleus together is greater than that required to hold a smaller nucleus together. However, the average energy per nucleon may be less. The least amount of energy per nucleon is required by intermediate nuclei, such as iron and copper, while very light nuclei require considerably more binding energy per nucleon (Figure 12-2-1).

The nuclear reaction can be written in short form. The example reaction equation, given earlier, could be expressed as $^{9}Be\ (\alpha,n)^{12}C$, or merely as the (α,n) for ^{9}Be. The former is preferred.

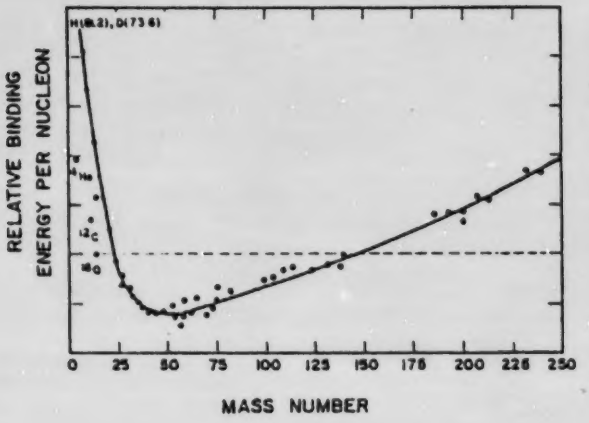


Figure 12-2-1. Average energy per nucleon.

TYPES OF NUCLEAR REACTIONS

Five basic nuclear reactions can occur with neutrons as the incident or "bombarding" particle:

Elastic scattering.
Inelastic scattering.
Neutron capture.
Particle ejection.
Fission.

Elastic scattering is a reaction in which kinetic energy is conserved. It is sometimes given the nickname of "billiard-ball collision" because the neutron may be thought of as bouncing off the target nucleus, much in the same manner as one billiard ball bounces off another. The neutron need not strike the target nucleus, but may be deflected by its nuclear field (Figure 12-2-2).

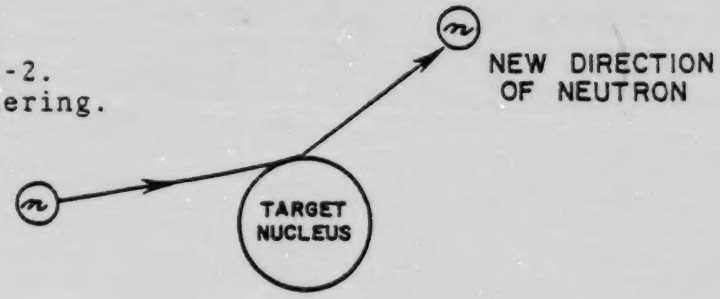


Figure 12-2-2.
Elastic scattering.

Elastic scattering is more apt to occur with fast neutrons than with slow ones. If the target nucleus is of the same size as a neutron (such as a hydrogen nucleus), it may be given enough kinetic energy to move (or be knocked out of) a chemical compound. This reaction is known as recoil nucleus (Figure 12-2-3). Proton recoil is an impor-

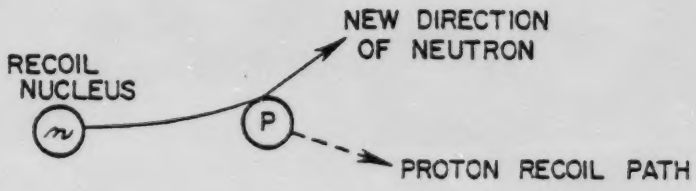


Figure 12-2-3.
Recoil nucleus.

tant method of fast neutron detection (discussed in the previous module).

Elastic scattering reactions are sometimes known as (n,n') reactions, signifying neutron in (n) at a given velocity and a neutron out at a different velocity (n').

Inelastic scattering is a process in which some kinetic energy is lost. The incident neutron may actually enter and pass through the target nucleus, transferring some of

its kinetic energy to the target nucleus. It may also accomplish this by passing very close (i.e., through the nuclear field) to the target nucleus. The energy imparted to the target nucleus puts it in a higher energy or excited state. When the target nucleus returns to the ground state, it emits this excess energy in the form of one or more gamma rays (Figure 12-2-4).

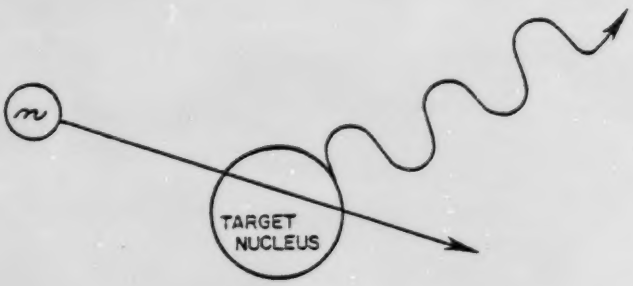
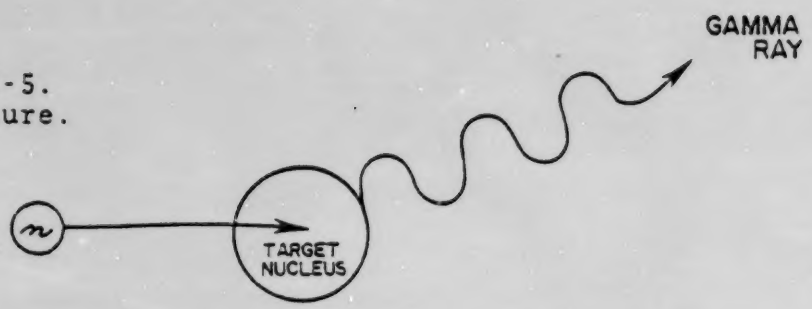


Figure 12-2-4.
Inelastic scattering.

The gamma ray is usually emitted instantaneously after the reaction, although some excited nuclei may be produced with relatively long half-lives. Inelastic scattering reactions of the type described are also (n,n') reactions and occur most often with intermediate and slow neutrons. It should be noted that, in both inelastic and elastic scattering, the incident particles (neutron and target nucleus) are the same as the product nuclei. There are no changes in the reactants except energy transfers.

Capture occurs when a neutron enters a target nucleus and remains there (Figure 12-2-5), producing a new nucleus with a mass number increased by 1:

Figure 12-2-5.
Neutron capture.



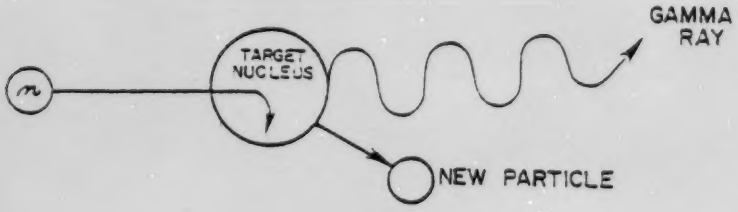
$$ {}^{1}_{0}n + {}^{59}_{27}Co \rightarrow {}^{60}_{27}Co + Q. $$

Eq. 12-2-3

The new nucleus, which is in an excited state, emits this excess energy as a gamma ray when it returns to ground state. Thus, capture reaction is sometimes called (n,γ) reaction. The new nucleus also may be radioactive, as in the example (Equation 12-2-3). Capture reactions are more apt to occur with thermal or slow neutrons than with fast ones.

Particle ejection occurs when a neutron enters a target nucleus and, because energy is imparted to the new nucleus, results in the emission of another particle (Figure 12-2-6). The nucleus remaining after emission of the par-

Figure 12-2-6.
Particle ejection.



ticle may be in an excited state, returning to the ground state by the emission of gamma radiation. The remaining nucleus may also be radioactive. A typical neutron reaction that produces particle ejection is shown by Equation 12-2-4.

$$^{1}_{0}n + {}^{10}_{5}B \rightarrow {}^{4}_{2}He + {}^{7}_{3}Li + Q. \qquad \text{Eq. 12-2-4}$$

In this reaction a neutron bombards a boron-10 nucleus, resulting in the emission of an alpha particle, and producing a lithium-7 nucleus (Figure 12-2-7). A compound nucleus,

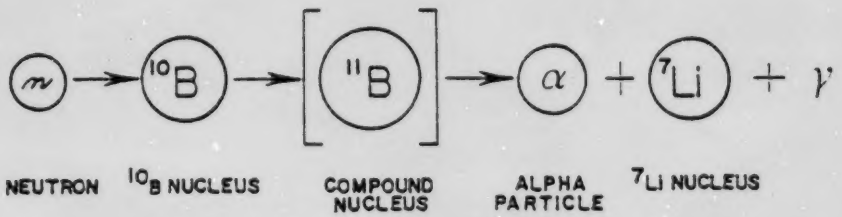


Figure 12-2-7. Particle ejection reaction, showing formation of compound intermediate nucleus.

formed as an intermediate stage, quickly decays to an alpha particle and a lithium-7 nucleus. Because it is so unstable, the compound nucleus may have a lifetime of microseconds or less.

Neutron fission occurs when a neutron causes a target nucleus to break into two nuclei, with the probable emission of additional neutrons and gamma rays (Figure 12-2-8). A neutron interacts with a ^{235}U nucleus to form a compound

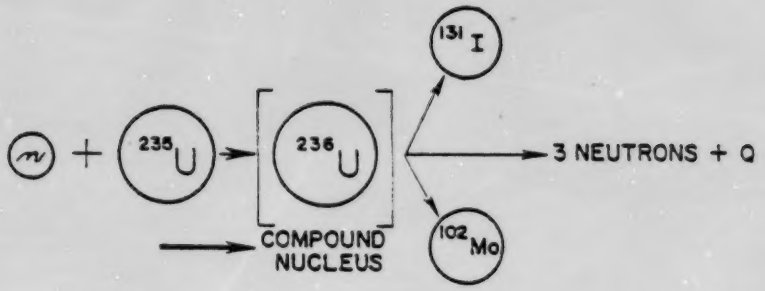


Figure 12-2-8. Typical fission reaction.

^{236}U nucleus, which almost instantaneously splits into two smaller (but still fairly large) nuclei (such as ^{131}I and ^{102}Mo), emitting three neutrons and gamma rays in the process. Both fission products are radioactive. (Fission is studied in detail in following modules.)

Other neutron reactions (in addition to the basic five) include spallation. This reaction, which occurs with very high-energy neutrons, is the breakup of a relatively light nucleus into small particles. For example, a ^{7}Li nucleus might be broken into an alpha particle, a proton and two neutrons. Certain elastic reactions other than the (n,n) reaction may also occur, usually with very high-energy neutrons.

NEUTRON INTERACTIONS WITH MATTER

As explained, any one of a number of possibilities of reactions can occur following bombardment of a material by a neutron, with some being more likely than others, depending on the specific situation. For example, when a ^{10}B

nucleus is bombarded by a neutron, the compound nucleus [^{11}B] formed can theoretically disintegrate or decay in one of several ways (Figure 12-2-9).

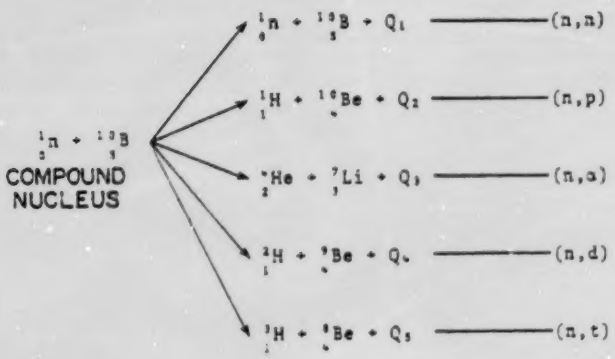


Figure 12-2-9. Possible [$^{11}_{5}B$] disintegration methods.

Breakup (decay) of a compound nucleus is related to total energy of the system. Nuclei liberating larger values of Q have a greater likelihood of taking place, while those with negative values of Q have to have at least that much energy supplied by a neutron for reaction to occur. Thus, if Q_2 and Q_4 are calculated, and Q_2 = 0.738 MeV while Q_4 = -4.36 MeV, a neutron must supply at least 4.36 MeV to Q_4 to make the (n,d) reaction occur. Otherwise, energy would not be conserved.

When Q is negative and energy must be supplied to make the reaction occur, the reaction is called endoergic. When Q is positive (i.e., energy is released), the reaction is termed exoergic.

The rule (the greater the Q, the more likely the reac-

tion) is only generally true. Other than energy considerations, various compensated aspects such as potential barrier of the nucleus and thermodynamics of the system need to be evaluated. The problem is highly complex and difficult, and the Rule of Q is adequate for most approximations.

CROSS SECTIONS

Cross section is a measure of the probability of an interaction. A neutron cross section is expressed in units of area, usually cm^2. Because the effective area of a nucleus for a particular nuclear reaction is small, a new unit of area was created to eliminate the use of large negative exponents. This unit "barn," is equal to 10^{-24} cm^2 - approximately the cross-sectional area of a typical nucleus.

Many factors influence a cross section:

Type of reaction.
Energy of incident neutron.
Material undergoing reaction.

Cross section, commonly expressed in barns per atom or microscopic cross section, is symbolized by σ. There is a microscopic cross section for each particular type of reaction.

Total cross section, the sum of all other cross sections (e.g., scattering, capture, fission, etc.), can be expressed mathematically:

$$\sigma_{tot} = \sigma_{scattering} + \sigma_{capture} + \sigma_{fission} + \cdots$$

or

$$\sigma_t = \sigma_s + \sigma_c + \sigma_f. \qquad \text{Eq. 12-2-5}$$

Certain types of cross sections tend to predominate at specific energies, and may also show recognizable trends as a function of energy. The greater the neutron energy, the lower the capture cross section:

$$\sigma_c \propto \frac{1}{v} \qquad \text{Eq. 12-2-6}$$

where

v = velocity of neutron.

There is a greater cross section with neutron velocity; consider the slower neutrons as spending more time in the vicinity of the nucleus and having proportionately more time to interact. Because the kinetic energy is given by $E = \frac{1}{2}mv^2$ (and $v = \sqrt{\frac{2E}{m}}$), capture cross section is related to energy:

$$\sigma_c \propto \frac{1}{v}$$

$$\sigma_c \propto \frac{1}{\sqrt{\frac{2E}{m}}} = \frac{1}{\sqrt{\frac{2}{m}} \times \sqrt{E}}. \qquad \text{Eq. 12-2-7}$$

But $\sqrt{\frac{2}{m}}$ is a constant, therefore, it does not affect proportionality. And cross section is a function of energy:

$$\sigma \propto \frac{1}{\sqrt{E}}. \qquad \text{Eq. 12-2-8}$$

Neutron cross sections often are plotted as a function of neutron energy for a particular nuclide. The plot is generally a log-log plot; that is, the log of the cross section (σ) is plotted against the log of the neutron energy (E). This arrangement permits several magnitudes of energies and cross sections to be shown on a relatively small, smoothed, hypothetical plot (Figure 12-2-10). In

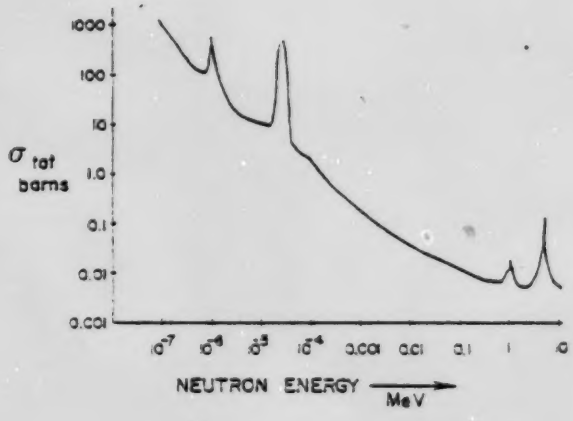


Figure 12-2-10.
Neutron cross section versus neutron energy.

general, the $\frac{1}{v}$ relationship seems to be followed. Irregularities appear at various points of the curve: 1) approxi-

mately 10^{-6}, 2) 3×10^{-5}, 3) 1 and 4) 5 MeV. (Neutrons at these specific energies have a much greater chance of interacting than do neighboring neutrons of slightly greater or lesser energies.) The sharp rises in cross section are known as resonances; the prime reactions that occur at these specific points are resonance reactions.

Resonances are a function of the energy levels within the nucleus. Some elements, such as boron, have few resonances and follow very closely the $\frac{1}{v}$ relationship over a wide energy range. Others, such as cadmium and indium, have many resonances; their cross section spectra are quite complex. For example, the capture cross section of cadmium shows a resonance for neutrons with energies about 0.176 eV. (At 0.176 eV, the cross section is 7200 barns. At 1 eV, the cross section is only 20 barns, and at 10 eV, about 4 barns.) Indium shows 3 resonance peaks between 1 and 10 eV, and 8 peaks between 10 and 100 eV (Figure 12-2-11). The largest of the cross-section peaks, at 1.4 eV, has a value of about 25,500 barns.

Indium and cadmium are good neutron absorbers, particularly for slow and thermal neutrons. Thus, they are sometimes used as thermal neutron shields, or as poisons to absorb neutrons in a reactor. Boron is also a good neutron absorber, but it has few resonances.

In general, capture cross sections predominate at low and intermediate energies, while particle reactions [(n,p), (n,α), etc.] tend to predominate at higher energies.

Neutron cross sections can be measured in various ways. The most common way is to insert a thin foil of very pure material in a beam of mono-energetic (single-energy) neutrons, then measuring, after a known exposure time, radioactive products in the foil. This method is known as neu-

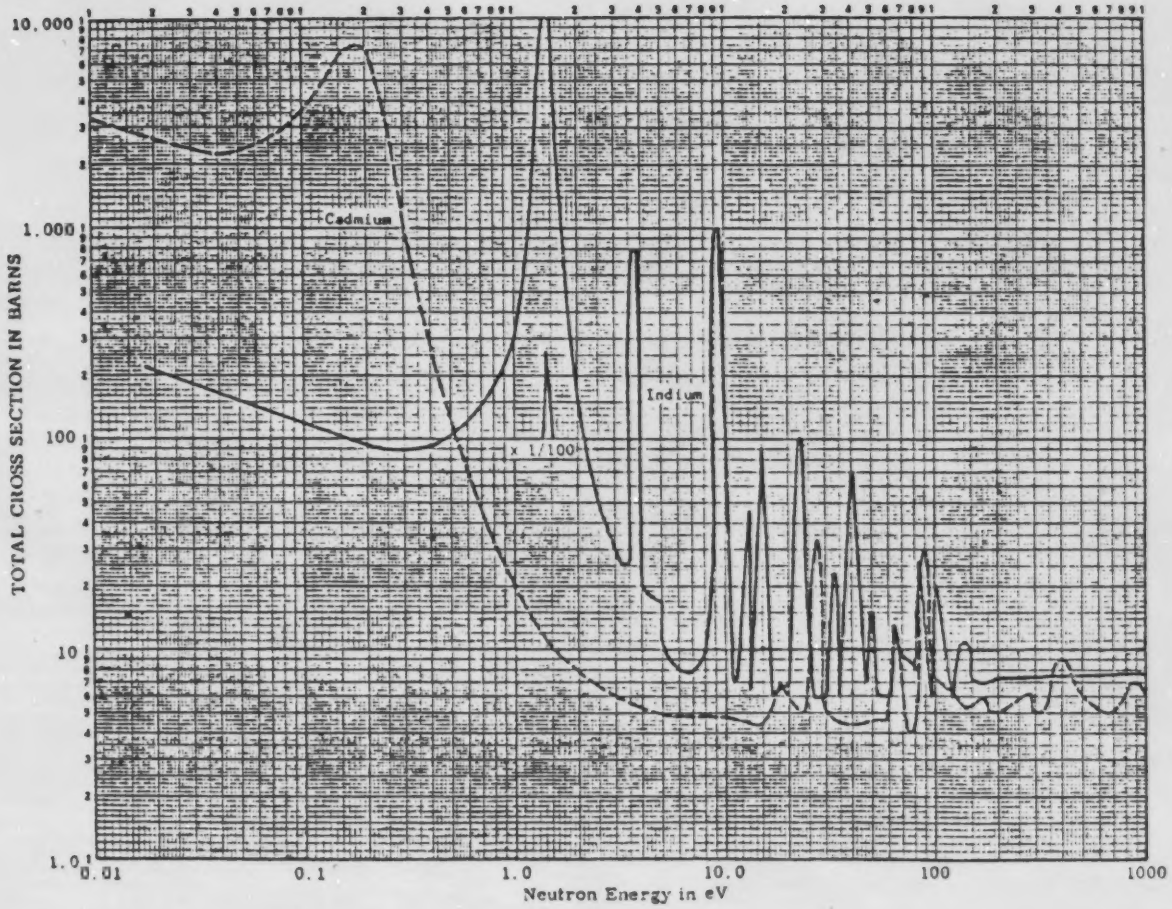


Figure 12-2-11. Typical neutron cross sections.

tron activation. Total cross section also can be obtained by direct measurement of the neutron beam (Figure 12-2-12). This total cross section, called removal cross section, removes neutrons from the beam, either by scattering or capture. Removal cross section, σ, can be calculated by

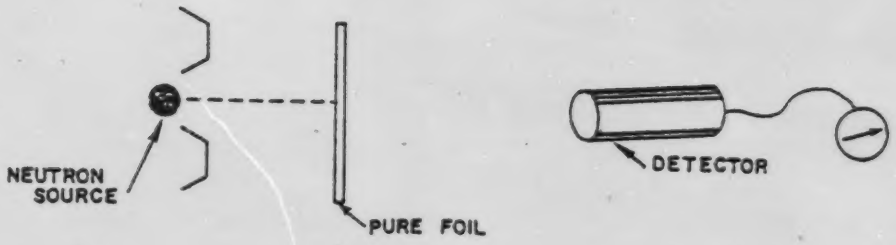


Figure 12-2-12. Neutron cross section measurement.

the relationship:

$$I = I_o d^{-N\sigma x} \qquad \text{Eq. 12-2-9}$$

where

I = flux after passing through foil.
I_o = flux (at same point) with no foil.
N = number of atoms per cm^3 in foil.
x = thickness of foil.

Equation 12-2-9 has the same form as the equation for attenuation of gamma rays and is used in much the same manner. As an illustration, consider the following problem:

EXAMPLE
Given: A sheet of gold, 0.01 cm thick, inserted

into a neutron beam, with a flux measurement of 640 n/cm^2-s; when foil is removed, flux-reading jumps to 880 n/cm^2-s. Also, the density (ρ) of gold is 19.32 g/cm^3 and its atomic weight is 197.

Find: Removal cross section N of gold.

Solution:

N = ρ x Avogadro's number ÷ atomic weight

$$= \frac{19.32 \text{ g/cm}^3 \times 6.023 \times 10^{23}}{197 \text{ grams/gram atomic weight}} \times \frac{\text{atoms}}{\text{grams atomic weight}}$$

$$= \frac{1.932 \times 10 \times 6.023 \times 10^{23}}{1.97 \times 10^2} \times \frac{\text{atoms}}{\text{cm}^3}$$

$$= 5.91 \times 10^{22} \text{ atoms/cm}^3.$$

Use Equation 12-2-9.

$$I = I_o e^{-N\sigma x}.$$

$I = 640 \text{ n/cm}^2\text{-s}.$

$I_o = 880 \text{ n/cm}^2\text{-s}.$

$x = 0.01 \text{ cm}.$

$$N = 5.91 \times 10^{22} \frac{\text{atoms}}{\text{cm}^3}.$$

$$640 = 880 e^{-5.91 \times 10^{22} \frac{\text{atoms}}{\text{cm}^3} \times \sigma \times 0.01 \text{ cm}}$$

Substitute and solve for σ:

$$\frac{640}{880} = e^{-5.91 \times 10^{22} \frac{\text{atoms}}{\text{cm}^3} \times \sigma \times 10^{-2} \text{ cm}}.$$

Take ℓn of both sides:

$$\ell n \propto \left(\frac{640}{880}\right) = -5.91 \times 10^{20} \frac{\text{atom}}{\text{cm}^2} \times \sigma$$

$$\ell n\ .727 = -0.319$$

$$= \sigma = \frac{-0.319}{-5.91 \times 10^{20}\ \text{atom/cm}^2}$$

$$\sigma = 5.39 \times 10^{-22}\ \text{cm}^2$$

$$\sigma = 539\ \text{barns}\ (1\ \text{barn} = 10^{-24}\ \text{cm}^2).$$

The calculation of N, the number of atoms per cm^3, can become complex and difficult if compounds or mixtures rather than single pure elements are involved. It sometimes is more convenient and useful, therefore, to use a different type of cross section - the so-called macroscopic cross section - to eliminate the need for calculating N.

MACROSCOPIC CROSS SECTION

A macroscopic cross section, symbolized by Σ, is mathematically equal to the product of N and σ:

$$\Sigma = N\sigma. \qquad \text{Eq. 12-2-10}$$

Thus, the macroscopic cross section has units of reciprocal (or inverse) length and is usually expressed as cm^{-1} or

"per centimeter." It may be thought of as the probability of a neutron interaction per unit length (or cm) of a material. In addition, there are individual macroscopic cross sections for capture (Σ_c), scattering (Σ_s) and fission (Σ_f).

The cross section, σ (called microscopic cross section to distinguish it from macroscopic cross section, Σ) refers to one atom. It is dependent only upon the material and neutron energy. Macroscopic cross section, on the other hand, refers to the cross section of one cm^3 of atoms. It depends on atomic density, on the different materials present, and on neutron energy.

An important application of macroscopic cross section is in neutron shielding calculations. Materials such as paraffin and polyethylene contain many elements and, rather than attempt to determine multiple microscopic cross sections, a macroscopic cross section is employed in these calculations.

EXAMPLE

Given: A hypothetical substance (polyplastin) is inserted in a neutron beam and the flux measured as 166 n/cm^2-s; when removed from the beam, the flux was measured as 283 n/cm^2-s; polyplastin is 3.6 cm thick.

Find: Macroscopic cross section of polyplastin.

Solution:

$$\Sigma = N\sigma$$

and

$$I = I_o e^{-N\sigma x}.$$

Substituting from Equation 12-2-10:

$$I = I_o e^{-\Sigma x}$$

$$166 = 283e^{-\Sigma} \times 3.6 \text{ cm}$$

Solving for Σ:

$$\frac{166}{283} = e^{-\Sigma} \times 3.6$$

$$\ln \left(\frac{166}{283}\right) = -3.6 \times \Sigma$$

$$\ln 0.59 = -3.6\Sigma$$

$$-0.534 = -3.6\Sigma$$

$$\Sigma = \frac{-0.534}{-3.6} = 0.15 \text{ cm}^{-1}.$$

e-FOLDING THICKNESS

A shielding term sometimes used, e-folding thickness, is the amount of shielding required to reduce neutron flux by a factor of e (the base of the natural logarithms, numerically equal to 2.718282). If a neutron beam is reduced by a factor of e, it is reduced to $\frac{1}{e} = 0.37$, or 37% of its original intensity.

Of perhaps more significance is the calculation of the cross sections for individual reactions. These are important for many reasons, one being that knowledge of these cross sections permits calculation of induced radioactivity (and hence dose rates).

Earlier in this section of the module, the cross section for a particular element in a neutron beam was calculated. What was calculated was σ_{tot}, the total microscopic cross section, which is the sum of the cross sections for all the individual neutron reactions:

$$\sigma_{tot} = \sigma_{scattering} + \sigma_{n,\alpha} + \sigma_{fission} + \sigma_{n,p} + \cdots$$

There are several ways to determine individual cross sections, but the easiest is by measuring one (or more) of the products of the reaction. Since these are usually radioactive, this measurement can often be made easily.

Suppose, for example that the cross section for a (n,α) reaction in aluminum is desired:

$$^{27}_{13}Al + ^{1}_{0}n \rightarrow ^{24}_{11}Na + ^{4}_{2}He + Q. \qquad \text{Eq. 12-2-11}$$

The product ^{24}Na is radioactive (having a 15-hour half-life and emitting two gammas per disintegration, one with an energy of 1369 keV and the other 2.754 keV). If only gamma rays are sought, then only the amount of ^{24}Na is measured, and not other radioactive species, such as ^{28}Al or ^{28}Mg, or the products of other possible reactions.

The equation used to calculate $\sigma_{(n,\alpha)}$ is derived from the equation used to calculate total removal cross section. But it has a somewhat different form because radioactivity

induced by neutrons is being evaluated (instead of a direct measurement of flux change).

To find out the (n,α) in aluminum, a small amount (say 1 gram) is placed in a neutron flux of 10^6 n/cm^2-s and left there for 100 minutes. It is then removed from the neutron flux and carried to the laboratory, where two hours later the radioactivity is measured from ^{24}Na as 410 disintegrations per second. Then $\sigma_{(n,\alpha)}$ is calculated as follows:

$$A = n\sigma\phi\ (1-e^{-\lambda\tau})e^{-\lambda t} \qquad \text{Eq. 12-2-12}$$

where

A = activity of sample, disintegration/second.
n = number of atoms.
σ = microscopic cross section, cm^2.
ϕ = neutron flux, neutrons/cm^2-s.
λ = decay constant.
τ = duration time.
t = time (removal from exposure until measurement).

Figure 12-2-13 gives a graphical presentation of build-up and decay. Note that, in the above calculation, all units of time were kept in seconds, and the equation is dimensionally correct:

$$\frac{\text{atoms (disintegrating)}}{\text{second}}$$

$$= \text{atoms} \times \frac{\text{cm}^2}{\text{n}} \times \frac{\text{n}}{\text{cm}^2\text{-s}}\ (1-e^{-s^{-1} \times s})\ (e^{-s^{-1} \times s})$$

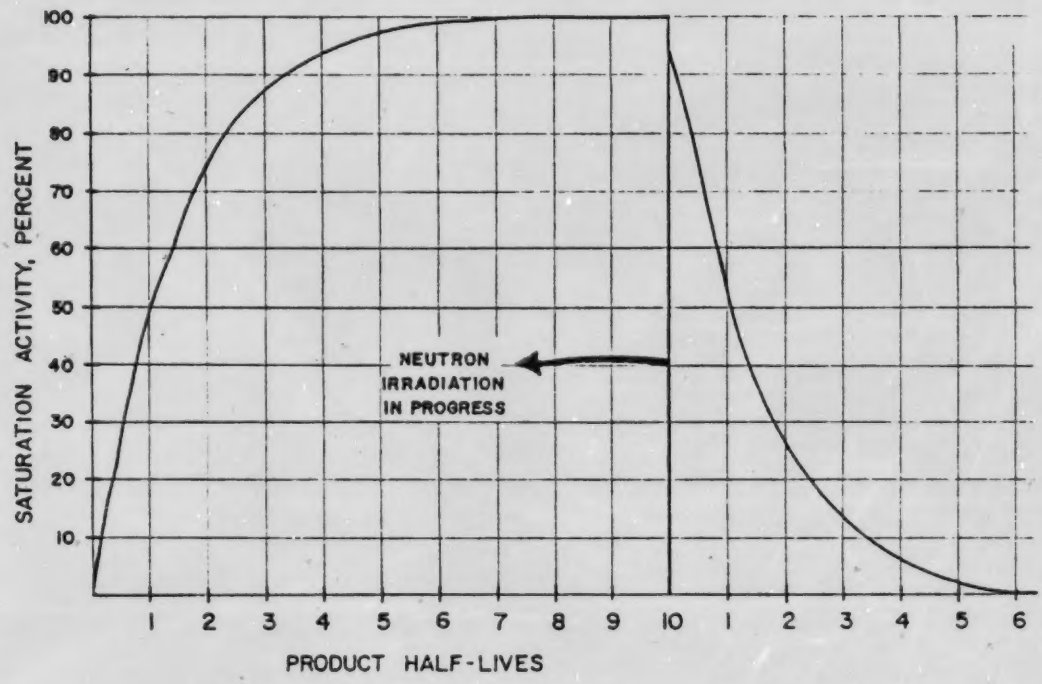


Figure 12-2-13. Relationship of activity buildup and decay of radioactive product as a result of neutron activation.

PRACTICAL APPLICATION OF CROSS SECTIONS

Generally, cross sections are known, and the above equations given are used frequently for neutron shielding calculations; values for σ_{tot} or Σ are available from hand-

books, most notably, Neutron Cross Sections, BNL-325, the so-called "barn book," which is a compilation of graphs of σ versus neutron energy for many elements and materials.

A most important application of cross sections is their use in calculating induced radioactivity from neutron bombardment, using Equation 12-2-12:

$$A = n\sigma\phi\ (1-e^{-\lambda\tau})\ e^{-\lambda t}.$$

EXAMPLE

Given: A source of ^{60}Co to be produced in a high flux research reactor; thermal flux is 10^{14} n/cm-s; ^{59}Co target mass is 10 g.

Find: How long one would have to expose the sample of ^{59}Co to obtain 1 Ci.

Solution: From a handbook or Chart of the Nuclides, find thermal neutron capture cross section of ^{59}Co. (For convenience, use 37.4 barns.)

1 Ci = 3.7×10^{10} atoms disintegrating per second, which is the value of A (in Equation 12-2-12). Number of atoms, n, is calculated as follows:

$$n = \frac{10 \text{ grams} \times 6.023 \times 10^{23} \frac{\text{atom}}{\text{g-at wt.}}}{59 \text{ grams/g-at wt.}}$$

$$= 1.02 \times 10^{23} \text{ atoms.}$$

The decay constant, λ, is calculated from the half-life of ^{60}Co:

$$\lambda = \frac{0.693}{5.35 \text{ yr} \times 3.15 \times 10^{7} \text{ s/yr}} = 4.1 \times 10^{-9} \text{ s}^{-1}$$

If distance is zero, to produce 1 Ci or 3.7×10^{10} dis/s at instant of removal from reactor, time after removal from reactor is zero; hence, $e^{-\lambda t} = e^{\circ} = 1$. Now, the only unknown is τ (irradiation time); values can be substituted in Equation 12-2-12 to solve for τ:

$$A = n\sigma\phi(1-e^{-\lambda\tau})\, e^{-\lambda t}$$

$$3.7 \times 10^{10} \text{ dps} = 1.02 \times 10^{23} \text{ atoms} \times 37.4 \times 10^{-24} \text{ cm}^2/\text{atoms} \times 10^{14} \text{ n/cm}^2\text{s} \times (1-e^{-4.1 \times 10^{-9}/\text{s}\ \tau})(e^{-4.1 \times 10^{-9} \text{ s} \times 0 \text{ s}}).$$

Therefore,

$$3.7 \times 10^{10} \text{ dps} = 3.81 \times 10^{14} \text{ atoms/s } (1-e^{-4.1 \times 10^{-9}\tau}) \times 1$$

$$9.71 \times 10^{-5} = 1-e^{-4.1 \times 10^{-9}\tau}$$

$$9.71 \times 10^{-5} - 1 = -e^{-4.1 \times 10^{-9}\tau}$$

$$\ln 0.999903 = -4.1 \times 10^{-9}\tau$$

$$\frac{-9.7 \times 10^{-5}}{-4.1 \times 10^{-9}} = \tau.$$

or

$\tau = 2.37 \times 10^4$ seconds
$= 3.95 \times 10^2$ minutes
$= 6.58$ hours.

An important concept related to cross section is that of albedo. Neutron albedo refers to reflected neutrons and is, therefore, related to the scattering cross section. Neutron reflectors are generally low-Z materials that are also good moderators. Such materials have high scatter cross sections and can reflect (or bounce) neutrons back into a reactor. Neutron albedo is often expressed as a fraction of the total neutron flux.

A final concept related to cross section is cadmium ratio. Cadmium has a very high capture cross section for thermal neutrons, in the neighborhood of several thousand barns. A crude but effective indication of how well a neutron flux or beam is thermalized can be obtained by making a measurement with a bare detector or foil and comparing it with a detector or foil that has been wrapped in cadmium. Usually, two gold or indium foils are used, one bare and one wrapped in a sufficient thickness of cadmium to remove 99+% of the thermal neutrons. A cadmium thickness of less than 0.001 inch is adequate to accomplish this. Hence, a bare foil and one wrapped in some cadmium (both foils with the same mass) can be placed in a neutron flux, exposed for the same amount of time, and radioactivity in each foil determined. The ratio of activation (i.e., radioactivity) in the bare foil to that in the cadmium-wrapped foil is the cadmium ratio:

$$\text{Cd ratio} = \frac{\text{activity in bare foil}}{\text{activity in Cd wrapped foil}}.$$

Eq. 12-2-13

REACTION RATE

Although there are many potential neutron reactions, the most important from a reactor standpoint are those collisions that reduce the energy of fast neutrons and thereby produce thermal neutrons. These reactions are primarily elastic scattering. When fast neutrons, having energies on the order of a few MeV, are introduced into a medium, such as water, which consists largely of atoms with nuclei about the same size as the neutron, a sequence of collisions between the neutrons and the nuclei occurs. With each collision, the neutrons lose energy and change their direction of travel (Figure 12-2-14). The x's refer to collisions with other (hydrogen) nuclei.

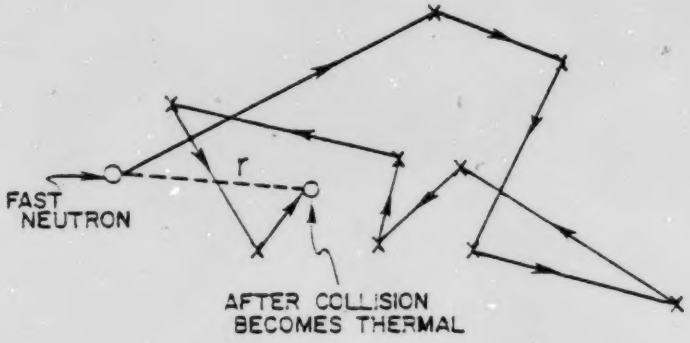


Figure 12-2-14. Typical path during slowing down of fast neutron. (Distance from point of origin to point of thermalization is shown by dashed line, r.)

The average elastic scattering collision is a function of two characteristics that depend only on the nucleus involved in the scattering:

The mean of the cosines of the angles of scattering.

The mean change in the natural logarithm of the energy of the neutron.

Each of these characteristics can be expressed mathematically, and the mathematical solutions demonstrate that light nuclei, such as hydrogen, tend to scatter neutrons in the forward direction and to provide the greatest energy loss per collision. Thus, a neutron passes through a series of collisions, traveling different distances between collisions and ultimately, after many collisions, stopping at some distance r from its point of origin as a thermal neutron.

This distance r, which is important in reactor work, is related to a concept known as neutron age. This age, given in units of area (actually, square centimeters), is related to the average distance (hence, time) between point of origin and point of thermalization. Obviously, the smaller the age, the better the material is at slowing down neutrons. Materials that slow down neutrons are called moderators; Table 12-2-1 shows the "age" for three commonly used moderators.

Neutron reaction rate R, a function of three variables, can be expressed mathematically:

$$R = \phi N \sigma \qquad \text{Eq. 12-2-14}$$

TABLE 12-2-1
AGE OF MODERATORS

Moderator	"Age" to Thermalization
Light water	26 cm^2
Heavy water	125 cm^2
Graphite	364 cm^2

where

R = reaction rate, reactions (or neutrons)/cm^3-s.

ϕ = neutron flux, neutrons/cm^2-s.

N = number of target atoms per cm^3.

σ = cross section per target atom, cm^2.

Obviously, the greater the neutron flux, the greater the reaction rate. Similarly, the greater the cross section or number of target atoms, the greater the reaction rate.

Note that reaction rate has units of n/cm^3-s (neutrons reacting per cubic centimeter per second). Also, reaction rate can be expressed in terms of Σ, the macroscopic cross section, because, as was shown above,

$$R = \phi\Sigma. \qquad \text{Eq. 12-2-15}$$

To illustrate the reaction rate, the microscopic fission cross section for ^{235}U is 681 b (for thermal neutrons), and the corresponding macroscopic fission cross section, $\Sigma_a = 32.7\ cm^{-1}$. In a small research reactor, where the thermal flux level is 4×10^{13} neutrons/cm^2-s in the center of the core, the reaction rate may be shown as follows:

$$R = \phi\Sigma = 4 \times 10^{13}\ \text{neutrons/cm}^2\text{-s} \times 32.7\ \text{cm}^{-1}$$
$$= 1.31 \times 10^{15}\ \text{n/cm}^3\text{-s}.$$

This rate is the same as that at which ^{235}U atoms are consumed, since they are transformed (or fissioned) by the reaction. Thus, reaction rate can be used to determine fuel consumption in a reactor.

EXERCISES

1. Write equations for the following reactions:
   - Neutron capture by ^{59}Co.
   - (n,α) or ^{14}N.
   - (n,2n) or ^{13}C.
2. Calculate Q for an (n,α) or ^{14}N reaction. Would this reaction take place with 2 MeV neutrons (according to the Rule of Q)?
3. Use Figure 12-2-11; determine resonance cross section for 38 eV neutrons in indium.

4. An accelerator produces a neutron beam with a flux of 3.6×10^{14} n/cm^2-s; when a sheet of indium is inserted in the beam, flux drops to 2.1×10^{14} n/cm^2-s; if indium is 0.6 mm thick, calculate macroscopic cross section for removal. What is its microscopic cross section?
5. Thermal neutron capture cross section for gold is 99 barns; a one-gram foil of gold is inserted into a reactor and exposed to a thermal neutron flux of 1.6×10^{12} n/cm^{-2}-s for three hours; it is removed and measured for radioactivity one week later. How much radioactivity would you expect to find?
6. A neutron shielding material has a macroscopic cross section for fission neutrons of 8.7 cm^{-1}. How many cm would be required to reduce a fission neutron flux of 6400 n/cm^2-s to 7.5 n/cm^2-s?

MATERIALS

1 neutron source.
Cadmium sheets and foils.
Gold foils.
1 neutron survey meter.
1 scaler and GM tube (counting equipment).
Paraffin or polyethylene sheets.
Semi-log paper.

NEUTRON ATTENUATION

Measure output from a neutron source with a survey meter. Plot change in measured output on semi-log paper, as several cadmium sheets of known thickness are successively inserted, one at a time, between source and survey meter. Calculate macroscopic cross section (for this particular source) for cadmium. Record all data on Data Table 12-2-1.

NEUTRON ACTIVATION

Place bare gold foil and a goil foil wrapped in cadmium equal distances from neutron source. Expose for an appropriate period (specified by instructor). With a laboratory scaler and GM tube, determine counting rates for bare and covered foils. Calculate Cd ratio. Also, calculate thermal capture cross section for gold (σ_c). Assume that all activation is caused by thermal neutrons. Record all data on Data Table 12-2-2.

Repeat this experiment with a sheet of polyethylene (or paraffin several inches thick) between source and foils. (Alternatively, foils may be imbedded in hydrogenous material.) Determine Cd ratio and explain why it varies from previous one.

DATA TABLE 12-2-1

Type neutron source _____

Model number of survey meter _____

Distance from source to survey meter _____

Thickness of Cadmium	Survey Meter Reading
0	

Directions on graph: Plot survey meter reading on vertical (semi-log) axis, and thickness on horizontal (linear) axis. Draw a straight line through data, extending line back to X = 0. From straight line, not actual data, pick off the following information:

I_o = _____, at X = 0.

I = _____, at X = _____.

(Pick some arbitrary I and a corresponding X.)

DATA TABLE 12-2-1 (continued)

Solve for Σ:

How do experimental values compare with accepted values (given by instructor)?

DATA TABLE 12-2-2

Type neutron source _____

Model number of scaler and tube _____

Mass of bare gold foil _____

Mass of covered gold foil _____

Activity in bare foil (net) _____

Activity in Cd covered foil (net) _____

Note: If foil masses are different, a correction must be made because activity is proportional to mass:

$$\text{Cd ratio} = \frac{\text{(activity in bare foil)}}{\text{(activity in Cd wrapped foil)}} \frac{\text{(mass of Cd wrapped foil)}}{\text{(mass of bare foil)}}$$

$$= \frac{(\quad\quad)}{(\quad\quad)} \frac{(\quad\quad)}{(\quad\quad)}$$

$$= \_\_\_\_\_.$$

Calculate thermal capture cross section:

How do experimental values compare with accepted values (given by instructor)?

DATA TABLE 12-2-3

Type neutron source _____

Model number of scaler and tube _____

Mass of bare gold foil _____

Mass of covered gold foil _____

Activity in bare foil (net) _____

Activity in Cd covered foil (net) _____

Note: If foil masses are different, a correction must be made because activity is proportional to mass.

$$\text{Cd ratio} = \frac{\text{(activity in bare foil)}}{\text{(activity in Cd wrapped foil)}} \frac{\text{(mass of Cd wrapped foil)}}{\text{(mass of bare foil)}}$$

$$= \frac{(\qquad\qquad)}{(\qquad\qquad)} \frac{(\qquad\qquad)}{(\qquad\qquad)}$$

$$= \_\_\_\_\_.$$

Calculate thermal capture cross section:

Explain why Cd ratio and thermal neutron cross section varies from previous case.

How do experimental values compare with accepted values (given by instructor)?

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1. List three different types of reactions that produce neutrons:

2. Write typical reaction equations for radioactive capture and charged particle emission:

3. Listed below are four neutron scattering reactions. Indicate process that takes place during reaction:

   Head-on collision with a small nucleus.

   Glancing collision with a small nucleus.

Head-on collision with a large nucleus.

Glancing collision with a large nucleus.

4. Briefly describe concept of cross section:

5. Why is cross section not equal to cross-sectional area of a nucleus?

6. Use "Chart of the Nuclides" and Radiological Health Handbook as references; define:
   Scattering.

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Absorption.

Fission cross sections.

7. Briefly explain differences between microscopic and macroscopic cross sections:

8. Use radionuclide and density; calculate macroscopic cross section for ^{12}C (find necessary values in "Chart of the Nuclides"):

9. Draw a typical absorption-versus-energy curve:

10. Briefly explain significance of major parts of curve drawn above:

11. Define:

Nuclear reaction.

Elastic scattering.

Inelastic scattering.

Capture.

Compound nucleus.

Fission.

Cross section.

Microscopic cross section.

$\frac{1}{v}$.

Resonance.

Macroscopic cross section.

Albedo.

Cadmium ratio.

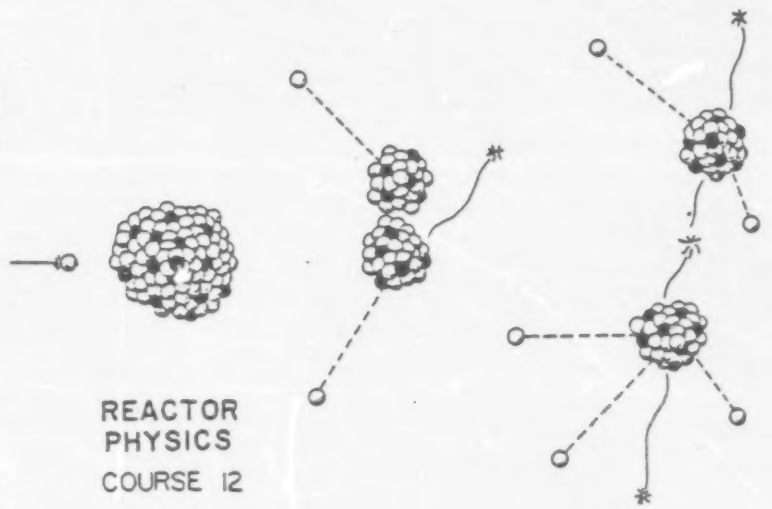
Neutron age.

Neutron reaction rate.

NUCLEAR TECHNOLOGY

MODULE 12-3 FISSION RATE AND REACTOR POWER. -

RONALD L. KATHREN AND LARRY B. CHURCH



TERC-SW

TECHNICAL EDUCATION RESEARCH CENTER — SOUTHWEST
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COURSE 12 REACTOR PHYSICS

MODULE 12-1 Neutron Physics

MODULE 12-2 Cross Sections and Reactions

MODULE 12-3 Fission Rate and Reactor Power

MODULE 12-4 Time Independent Nuclear Theory

MODULE 12-5 Time Dependent Nuclear Theory

MODULE 12-6 Reactivity - Fuel Moderator and Control Systems

MODULE 12-7 Internal Effects on Reactivity

MODULE 12-8 Reactor Operating Characteristics

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INTRODUCTION

When certain heavy nuclei capture neutrons, the nuclei split into smaller nuclei. This reaction is called "fission." In addition to the two main fission products (smaller nuclei), two other products result from the fission process: 1) energy is released and 2) usually, two or three neutrons are released. These neutrons may then interact with other nuclei to cause additional fission, and a process (known as a chain reaction) ensues.

This module is devoted to studying the energy (or heat) and power produced from such nuclear fission.

MODULE PREREQUISITES

The student should have completed Module 12-2 of this Course, along with its prerequisites.

OBJECTIVES

Upon completion of this module, the student should be able to:

1. In one or two sentences, describe the important contributions of each of these men to the discovery of

the fission process: 1) Fermi, 2) Hahn and 3) Strassman.

2. State why it is easier to cause fission with neutrons than with other nuclear projectiles.
3. Differentiate between fissionable, fissile and fertile materials.
4. List the three most common fissionable nuclides.
5. Write the nuclear equations for converting ^{238}U into ^{239}Pu and ^{232}Th into ^{233}U.
6. Draw a curve for fission cross section versus neutron energy for ^{235}U.
7. Explain why most fission fragments are β^- emitters.
8. Draw and explain the significance of the fission yield curve for ^{235}U.
9. Calculate energy released in fission, given identity and masses of reactants and products.
10. Explain why the energy of a neutrino is not usable in a power reactor.
11. Discuss distribution of fission energy between the three major fission products.
12. Verify that 3.1×10^{10} fissions per second will produce 1 watt.
13. Given that 280 ^{235}U atoms spontaneously fission per second per ton of material, calculate how many tons are required to produce one watt of power from spontaneous fission.
14. Find the power at any time after shutdown, given a curve or equation relating the power and time after shutdown from full operating power.
15. Define:

    Transuranium elements.

    Fissile.

    Fertile materials.

87

Fission yield.

Unsymmetrical yield.

Decay heating.

Spontaneous fission.

16. Calculate mass and volume of all fission products generated after a 1000-MW reactor has been operational for one year.

DISCOVERY OF FISSION

In the mid-1930's, Enrico Fermi, working in Italy, exposed uranium (Z = 92) to low-energy neutrons (thermal or slow neutrons) with the intention of producing transuranium elements (Z > 92), which would be created by the (n,γ) reaction on the uranium followed by subsequent β^- decay. There were many different observed products, and this fact tended to confuse Fermi and others working on this approach. Then, in 1938, Otto Hahn and F. Strassman, German scientists, convincingly showed by chemical separation that most of the products were not transuranic elements, but rather were very much lighter than uranium with atomic numbers around 35 to 65. Clearly, for this to occur, the heavier uranium atom must have split (i.e., undergone fission).

Shortly thereafter, it was also observed that several neutrons were emitted as a part of the overall fission process. If the neutrons could be used to bombard more uranium, then more fission would occur and a chain reaction would ensue (Figure 12-3-1).

By 1942, Fermi had emigrated to the United States. On December 2, 1942, he and his coworkers at the University of Chicago demonstrated the first self-sustaining chain nuclear reaction under the stands at Stagg Field. The fuel was 40 tons of natural uranium mixed with blocks of graphite. This configuration resembled a pile of bricks and was given the name of a nuclear "pile," rather than nuclear "reactor," which is used today.

Shortly after the discovery of fission it was realized that, when uranium fissions, a considerable amount of energy

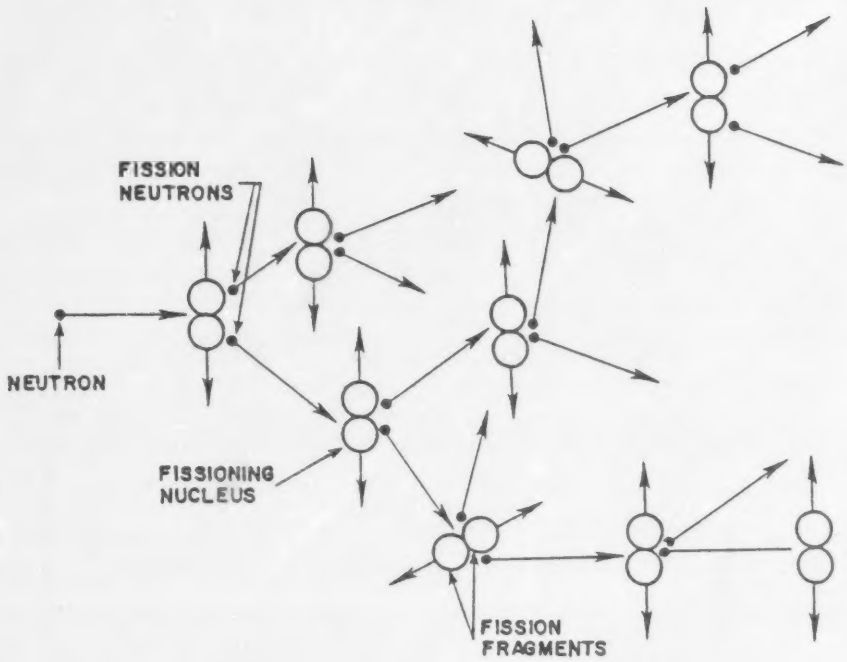

Figure 12-3-1. Fission chain reaction.

is released. The military significance of this energy was quickly realized by both the U.S. and German governments. The U.S. reaction was a successful crash program, called the Manhattan Project, to build a nuclear weapon.

So far, the major elements of the fission process have been outlined; that is, the two major inputs to the process have been identified (Figure 12-3-2): 1) a slow neutron and 2) a suitable heavy nucleus.

The three major products are 1) fission fragments, 2) neutrons and 3) energy. Let us examine each of these two processes and three products in much more detail.

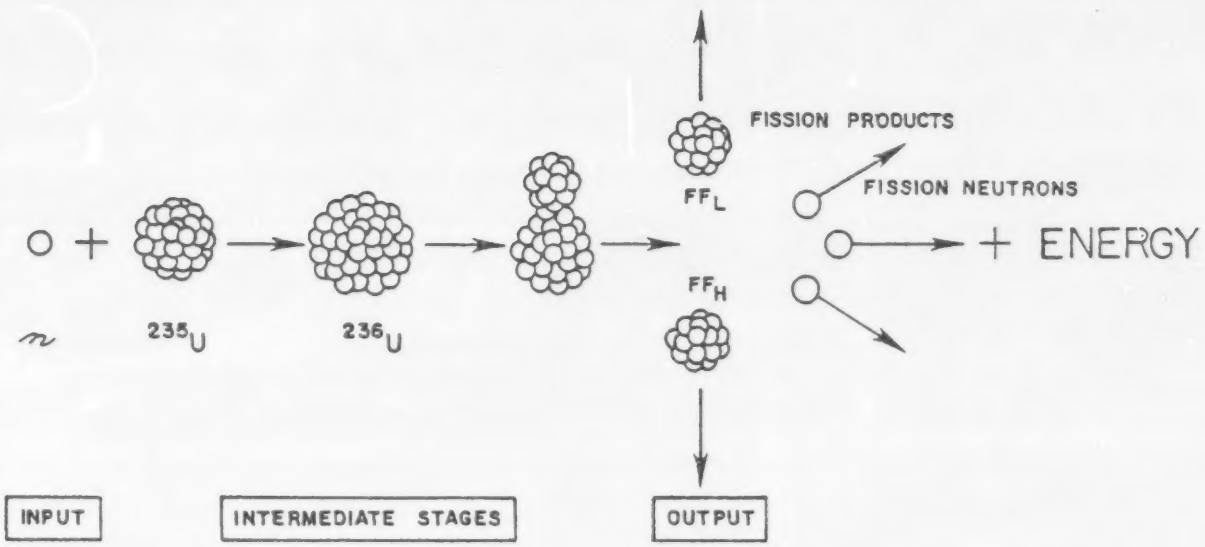

Figure 12-3-2. Four stages in fission of ^{235}U. (FF = fission fragments; H and L = heavy and light.)

FUEL

Only three nuclides exhibit the needed properties (of being both heavy and having an odd mass number) to be able to fission following the capture of a thermal neutron; three are ^{235}U, ^{233}U and ^{239}Pu. These nuclides, because they exhibit these special properties, are classified as fissile. Of the three, only ^{235}U is found in nature. The other two must be made from other nuclides. These other nuclides are classified as fertile materials (e.g., ^{238}U, ^{232}Th) because they exist in nature and fissionable materials can be made from them. Equations 12-3-1 and 12-3-2 give the pathways for the two transformations:

$$^{238}U\ (n,\gamma)\ ^{239}U \xrightarrow[\beta^-]{} {}^{239}Np \xrightarrow[\beta^-]{} {}^{239}Pu.$$

Equation 12-3-1
(Equation = Eq.)

$$^{232}Th\ (n,\gamma)\ ^{233}Th \xrightarrow[\beta^-]{} {}^{233}Pa \xrightarrow[\beta^-]{} {}^{233}U.$$

Eq. 12-3-2

An important point to be realized from the above discussion is that the only naturally occurring fissile fuel is ^{235}U. It occurs in nature with an isotopic abundance of ~ 0.72%. The nonfissile, but fertile ^{238}U has an isotopic abundance of ~ 99.27%. A large fraction of the effort of the Manhattan Project was to separate and concentrate the ^{235}U from the ^{238}U.

All of this discussion is not meant to imply that ^{238}U or ^{232}Th will not fission. They will. But the necessary neutron energy must be of the order of 1 MeV. Therefore, these nuclides are classified as "fissionable." (They are not fissile because the energy spectrum of their fission neutrons is such that not all have sufficient energy to allow further fission and sustain a chain reaction.)

To summarize: A fissile nuclide undergoes fission, releasing neutrons, which can cause more fission. A fissionable nuclide can undergo fission, but will not necessarily sustain a chain reaction. A fertile nuclide, by definition, can be transformed into a fissile nuclide.

The interaction of a neutron with a uranium atom can occur at any neutron energy. The neutron is unlike any charged particle, such as a proton or alpha, which must

have enough energy to overcome the coulombic barrier created between the charge of the heavy nucleus (92 protons in the case of uranium) and the charged particle. As will be shown in the next section, it is also highly desirable to have the incident neutrons at as low an energy as possible.

THERMAL NEUTRONS

The fission cross section, σ_f in barns, as a function of the neutron energy in MeV (Figure 12-3-3) is an excita-

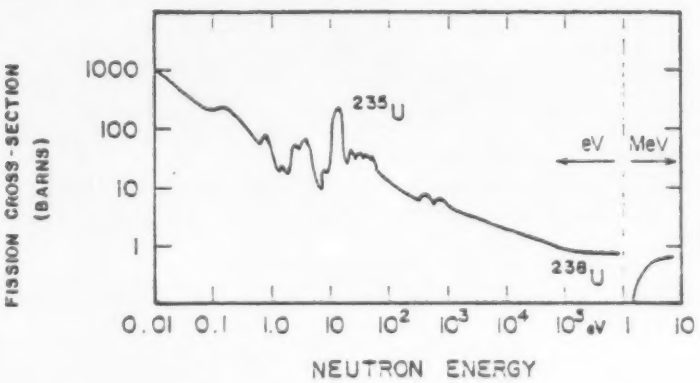

Figure 12-3-3. Fission excitation function of ^{235}U.

tion function. As can be seen, the lower the energy, the more likely it is that fission will occur. As is shown, a reduction in energy by a factor of 10 increases the fission cross section by about 3.3 times. The irregular portion of the excitation function around 10 eV is attributed

to a phenomenon known as resonance absorption. Within this energy region, the neutron can easily fill excited energy levels in the ^{235}U that are not populated at lower neutron energies. Thus, the reaction cross section quickly rises and falls within the population of neutrons into these specific excited energy states.

Once the fissile nuclide has captured a neutron, it can de-excite by either gamma-ray emission or fission. Of the two pathways shown (Figure 12-3-4), only the fission route will be discussed here.

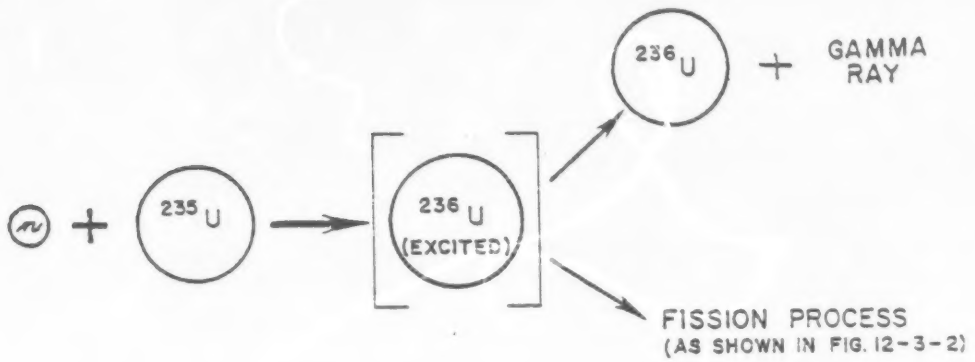

Figure 12-3-4. Showing the competitive paths following capture of a neutron by ^{235}U.

FISSION FRAGMENTS

For the reaction shown earlier (Figure 12-3-2), fission fragments are spread between mass numbers 70 and 160,

with a spread in atomic numbers between 35 and 65. While this spread occurs when many fission events are investigated, any one individual event must have products with a mass, including the two or three fission neutrons, of 236 and whose atomic number sum is 92. Thermal-energy induced fission usually creates unequal fission fragment masses. The mass ratio of the heavy-to-light fragment is ~ 3 to 2. In addition, neutron-to-proton ratio (N/Z) of the fission fragments reflects the (N/Z) ratio of the fissile parent. As a result, most fission products from ^{235}U have (N/Z) ratios around (235-92)/92 = 143/92 = 1.55.

Given these constraints, a typical, but by no means unique, set of fission fragments from ^{235}U would be ^{97}Sr and ^{137}Xe. The entire nuclear equation thus would be:

$$^{1}_{0}n + ^{235}_{92}U \rightarrow [^{236}_{92}U] \rightarrow ^{97}_{38}Sr + ^{137}_{54}Xe + 2n.$$

Eq. 12-3-3

These fission fragments are generally radioactive and decay through specific chains to stable isotopes.

The nuclear stability curve (Figure 12-3-5) shows that the common fragments from the fission of ^{235}U (N/Z ≈ 1.55) rest above the stability line in the region of β^- decay. By this decay, N/Z ratio of the fission fragments is decreased to a point along the line of stability where further decay will not occur. The following example illustrates the process leading to stability and the relationship with the N/Z ratio.

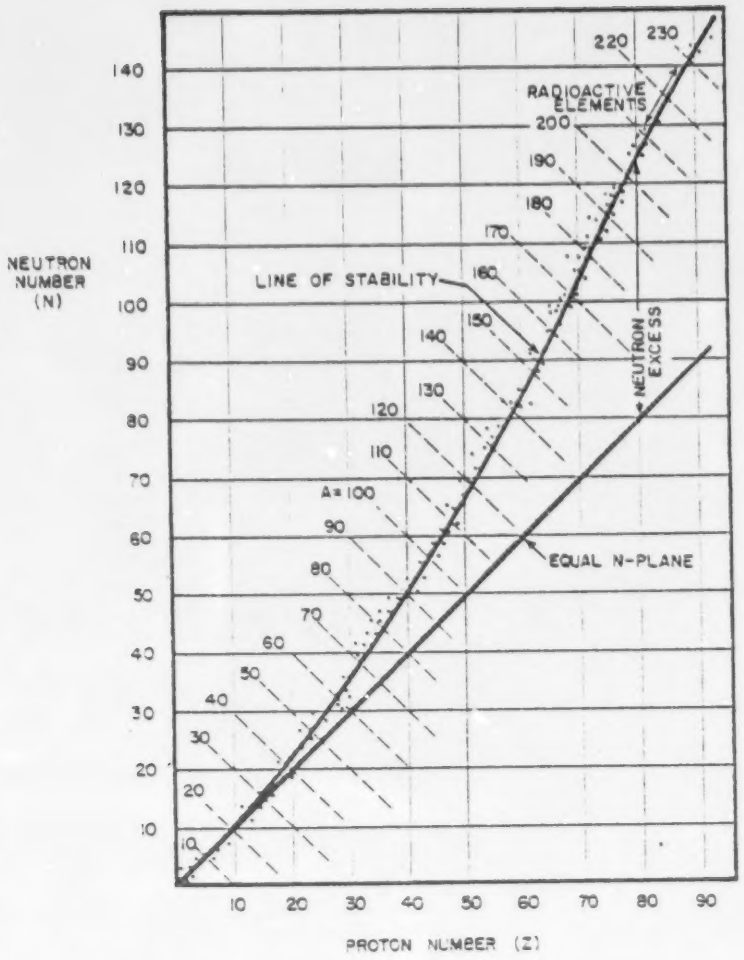

Figure 12-3-5. Nuclear stability curve.

EXAMPLE

$^{137}_{52}Te \xrightarrow{\beta^-} {}^{137}_{53}I \xrightarrow{\beta-} {}^{137}_{54}Xe \xrightarrow{\beta-} {}^{137}_{55}Cs \xrightarrow{\beta-} {}^{137}_{56}Ba$ stable

N/Z ratio =

1.63 1.58 1.53 1.49 1.44

It is important to realize that there are two different numbers associated with the fission yield of each fragment nuclide. The independent yield is that resulting directly from fission; in other words, that which is formed at time of fission. Cumulative yield, on the other hand, of a fission fragment is the sum of the independent yield plus the yield resulting from any β^- decay, and is time dependent. In the example, 21% of the ^{137}Xe yield from ^{235}U fission is independent. The remainder (79%) comes via the β^- decay of ^{137}I.

A common way of presenting the yields of all the products is to plot the cumulative fission yield for the entire mass chain as a function of the mass number. If, in the above example, the cumulative fission yield for ^{137}Xe would be plotted at 137, a fission yield curve for thermal neutrons interacting with ^{235}U may be developed (Figure 12-3-6).

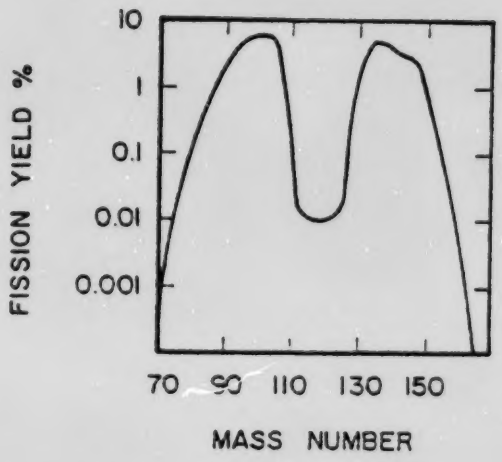

Figure 12-3-6.
Fission yield curve for ^{235}U interacting with thermal neutrons.

The major points of interest in this graph are summarized as follows:

Unsymmetrical yield. One fission fragment is almost always larger than the other, by a radio of about 3:2.

The mass numbers of the fission products range from about 70 to 160.

Maximum yields of about 9% occur for mass numbers of 95 and 135. (Yields are independent of whether the neutrons are thermal or 14 MeV, as would come from a D-T source.)

FISSION NEUTRONS

As described above, neutron energy distribution is an important parameter in a reactor. As neutrons are created in fission, their most probable energy is about 0.5 MeV, with the average energy near 2.0 MeV. Neutrons are produced with energies in the MeV range (Figure 12-3-7), but the probability for fission is greatest with energies

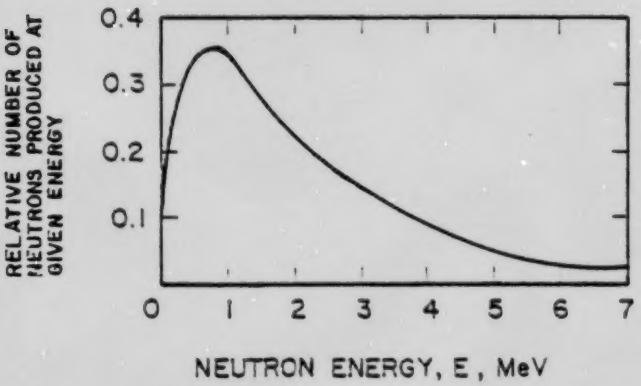

Figure 12-3-7.
Neutron energy spectrum covered by fission.

in the eV range (Figure 12-3-3); therefore, for most efficient operation, the neutrons must be slowed down or reduced in energy.

The process of slowing down the neutrons is called "moderation." A comparison of Figure 12-3-7 with 12-3-3 shows why moderation is necessary for efficient neutron use.

The average number of neutrons produced, $\bar{\nu}$, per event depends upon the fissile material and is summarized in Table 12-3-1.

TABLE 12-3-1
NUMBER OF NEUTRONS PRODUCED PER FISSION

Nuclide	$\bar{\nu}$
^{235}U	2.44
^{239}Pu	2.90
^{233}U	2.50

FISSION-PRODUCED ENERGY

The last, and perhaps most useful and important, fission product to be discussed is energy. To understand the source of this energy release, it is necessary to realize

the interrelationship equating mass, m, and energy, E.

Energy is produced in the fission process by transformation of a minute amount of mass into energy according to Einstein's equation:

$$E = mc^2 \qquad \text{Eq. 12-3-4}$$

where

E = energy, ergs.
m = mass loss or gain, grams.
c = velocity of light ~3 x 10^{10} cm/s.

EXAMPLE

Energy released from typical fission reaction fission fragments:

^{97}Sr (at. mass = 96.923 amu).
^{136}I (at. mass = 135.907 amu).

No. of neutrons released = 3.

Reactants:

neutron	1.009	amu
^{235}U +	235.044	amu
	236.053	amu (total mass of reactants).

Products:

^{97}Sr	96.923	amu
^{136}I	135.907	amu
3 neutrons	3.027	amu
	235.857	amu (total mass of products).

Difference in atomic mass units:

Reactants - products: 0.196 amu;

therefore, m = 0.196 in the equation $E = mc^2$ and

$$E = mx \left(931.2 \frac{MeV}{amu}\right)$$

Energy released = 0.196 amu x (931.2 MeV/amu).

E = 182 MeV per fission process.

The exact energy per fission depends on the specific products selected. The rounded-off number most often used is 200 MeV per fission; therefore, this number is used in the following calculations.

Given that power has units of energy per time, it is now possible to calculate the fission rate necessary to generate a certain power.

EXAMPLE

Given: Power = 10 MW.

Useful constants:

Energy ~ 200 MeV/fission.

1 MeV = 1.6×10^{-6} ergs.

1 erg/s = 10^{-7} watt.

Find: Fission rate (fissions/second) necessary to produce 10 MW of thermal power.

Solution:

Step 1. Convert 10 MW to ergs/s:

$$10 \text{ MW} = 10 \text{ MW} \times \left(\frac{10^6 \text{ watts}}{\text{MW}}\right) \times \left(\frac{1 \text{ erg/s}}{10^{-7} \text{ watt}}\right)$$

$= 10^{14} \frac{\text{ergs}}{\text{s}}$.

Step 2. Convert ergs/s to MeV/s:

$$10^{14} \frac{\text{ergs}}{\text{s}} \times \left(\frac{1 \text{ MeV}}{1.6 \times 10^{-6} \text{ ergs}}\right)$$

$$= 6.25 \times 10^{19} \frac{\text{MeV}}{\text{s}}.$$

Step 3. Convert $\frac{\text{MeV}}{\text{s}}$ to fissions/s:

$$6.25 \times 10^{19} \frac{\text{MeV}}{\text{s}} \times \left(\frac{1 \text{ fission}}{200 \text{ MeV}}\right)$$

$$= 3.125 \times 10^{17} \frac{\text{fissions}}{\text{second}}.$$

Thus, the generation of 10 MW of thermal power requires 3.12×10^{17} fissions per second.

ORIGINS OF FISSION ENERGY

It has been shown above that the ultimate source of the energy is the mass difference between the products and the reactants. A point now to be considered pertains to how and when, during the fission reaction, is energy released, and to what extent is it a usable heat source. About 84% of the energy from the total fission process is kinetic (usable) energy of the fission fragments. Another 6% of a 200-MeV/fission occurs simultaneously with the fission process. It is in the form of kinetic energy of the neutrons and gamma rays emitted during fission. Both of these are also slowed down by the moderator and/or struc-

tural materials surrounding the reactor, and this energy is also usable as heat. The remaining 10% of the 200-MeV/fission is released after fission by radioactive decay of the fission products, generally during the β^- decay of the fission product chains. The majority of β^- particles and γ rays emitted during decay are captured within the core, and this energy too is recoverable as heat. Approximately 12 MeV (6%) per fission is emitted as neutrinos during the decay. Because of their low mass, neutrinos are not stopped within the reactor core and their energy is consequently lost as useful heat. A small source of energy results when neutrons, which do not cause fission, are captured via the (n,γ) reaction. The gamma rays released from these reactions, like the prompt fission γ rays, are stopped within the reactor core. Table 12-3-2 summarizes energy emitted and recovered per fission of ^{235}U.

TABLE 12-3-2

EMITTED AND RECOVERABLE ENERGY FROM FISSION OF A ^{235}U ATOM

	Emitted Energy (MeV)	Recoverable Energy (MeV)
Fission fragments	168	168
Fission neutrons	5	5
Prompt γ radiation	7	7
Fission product decay		
β radiation	8	8
γ radiation	7	7
Neutrinos	12	0
Capture γ radiation	0	~5
Total	207	~200

The fact that the fission products deliver up to 10% of the total energy per fission at a time well after fission shows why the power of a reactor does not drop instantly to zero the moment the chain reaction ceases. This 10% heat release occurs after the primary fission process and contributes to post-shutdown or decay heating. Because the fission products are radioactive, the rate of heating diminishes with time and is inversely proportional to the half-life of the average fission product.

A typical decay heating curve (Figure 12-3-8) assumes

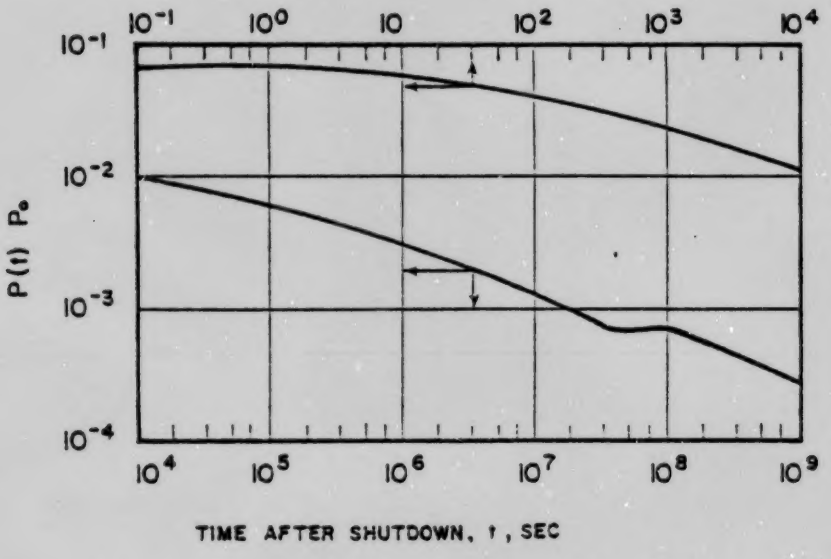

Figure 12-3-8. Ratio $P(t)/P_o$ of the decay heating to reactor operating power as a function of time, t, after shutdown.

the reactor has been operating long enough to build up an equilibrium concentration of the fission products. The time axis spans 10^{-1} second to 10^{9} seconds (about 30 years).

The vertical (power) axis is given as the ratio of the power at time t, P(t), to the full operating power, P_o.

A mathematical approximation of this curve is given by Equation 12-3-5, where t is expressed in days after shutdown following reactor operation of several months:

$$\frac{P(t)}{P_o} = (6.1 \times 10^{-3})\ (t^{-0.2}). \qquad \text{Eq. 12-3-5}$$

For example, the calculation of how much power remains in a 10-MW reactor one day after shutdown can be determined from either Figure 12-3-4 or Equation 12-3-5. From Figure 12-3-4, one day is 8.6×10^4 seconds and $P(\text{one day})/P_o = 5 \times 10^{-3}$. Thus, one day after shutdown 10 MW x 5×10^{-3} = 50×10^{-3} MW, or 50 kW of power remains. From Equation 12-3-5, $P(\text{one day})/P_o = 6.1 \times 10^{-3}\ (1)^{-0.2} = 6.1 \times 10^{-3}$ and the power after one day is computed to be 61 kW. Of these two answers, the one taken from the graph is probably the more accurate because one simple estimating equation cannot closely follow the entire curve.

SPONTANEOUS FISSION

So far, this module has discussed only particle induced fission, for example, thermal neutrons interacting with ^{235}U. The student should be aware, however, that some heavy nuclides occasionally undergo spontaneous fission (one of

the modes of radioactive decay). For example, about 3.5 x 10^{-7}% of the time ^{235}U undergoes spontaneous fission. The rest of the time it decays. Because of its low probability of occurrence, spontaneous fission is not significant in this study of neutron induced fission.

EXERCISES

1. Estimate the power of a 1000-MW reactor at the following times after shutdown of a very long run:
   - 1 hour.
   - 1 day.
   - 1 week.
   - 1 month.
   - 1 year.
   - 10 years.
2. Calculate the fissions per second to yield 1 watt.
3. Calculate the matter converted into energy via fission per second to yield 1 watt.
4. Write two fission reactions that may occur in a uranium reactor that has been operational for a while. (Use FF1 and FF2 to symbolize the two fission fragments.)

MATERIALS

Slide rule or hand calculator.
Handbook of Chemistry and Physics
Sealed uranium source.
Energy-calibrated multichannel analyzer and Ge(Li) detector.
Source of neutrons.
G.E. Chart of the Nuclides.
Semi-log paper.

PROCEDURES

1. Calculate mass of all fission products present in a 1000-MW (thermal) reactor after it has run continuously for one year:
   a. Calculate the seconds in a year _____(1).
   b. From second EXAMPLE, calculate fission per second required to give 1000-MW thermal _____(2).
   c. In one year _____(2) fission/s x _____(1) s/year = _____(3) fission/year.
   d. For each fission, one heavy and one light mass fission product will be formed. From Figure 12-3-6, determine most probable mass for the heavy and light product: $[\frac{amu}{atom} = \frac{grams}{moles}]$:
      _____(4) g/mole = M(heavy);
      _____(5) g/mole = M(light).

e. Calculate mass of the light mass products:
_____(3) fission/year x _____(5) g/mole x 1 atom/fission =

$$\frac{\text{\_\_\_\_\_}}{6.023 \times 10^{23} \text{ atom/mole}} =$$

_____(6) g/year of light mass product.

f. Similarly to "e" above, calculate mass of the heavy mass product:
_____(3) fission/year x _____(4) g/mole x 1 atom/fission

$$= \frac{\text{\_\_\_\_\_}}{6.023 \times 10^{23} \text{ atom/mole}} = \text{\_\_\_\_\_}(7) \text{ g/year}$$

of the heavy mass product.

g. Total mass/year: _____(6) + _____(7) = _____g/year.

2. Calculate volume of the fission product whose mass was calculated in Procedure 1 above.
For each fragment mass chosen in Procedure 1 above, and using the Handbook of Chemistry and Physics and the G.E. Chart of the Nuclides, fill in Data Table 12-3-1.

Note: Realize that the above calculations for Procedures 1 and 2 have assumed that all the fission fragments with compounds have densities of (7) g/cm^3, etc. Obviously, this is a generalized solution to the problem. To solve problems 1 and 2 exactly requires treating each mass separately, knowing exactly what compounds will be created at the reprocessing plant and then adding the weights and volumes for each element and compound to get a more exact solution for 1 and 2 respectively.

DATA TABLE 12-3-1

Description	Heavy Mass Product	Light Mass Product
1. Mass of fragment	_____(d) gram/mole	_____(e) gram/mole
2. The longest-lived radionuclide in the β^--decay chain with the mass in 1 above	_____	_____
3. A common compound of the element of 2 above; such a compound would be an oxide or sulfide	_____	_____
4. The molecular weight of the compound of 3 above	_____	_____ gram/mole
5. The mass fraction of element 2 above in compound 3 above	_____atoms molecule^{-1}. _____(d) g mole^{-1} _____ _____(4) g mole^{-1} • _____	_____atoms molecule^{-1}. _____(d) g mole^{-1} _____ _____(4) h mole^{-1} • _____
6. The mass per year of the fission fragment compound	_____(g) g year^{-1} _____ _____(5) • _____ g year^{-1}	_____(f) g year^{-1} _____ _____(5) • _____ g year^{-1}
7. Density of compound 3 above	_____ g cm^{-3}	_____ g cm^{-3}
8. The volume per year associated with each fragment	_____(6) g year^{-1} _____ _____(7) g cm^{-3} • _____cm^3 year^{-1}	_____(6) g year^{-1} _____ _____ g cm^{-3} • _____cm^3 year^{-1}
9. Total volume per year • _____(8) cm^{3}year^{-1}(Heavy • _____(8) cm^{3}year^{-1}(Light) • _____ cm^3 year		

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Grant, P.M.; Miller, G.E. and Rowland, F.S. Gamma Transition Energies of the Major Fission Products, Report No. 1971-1. Irvine, CA: Nuclear Reactor Facility at University of California, 1971.

LaMarsh, John R. Introduction to Nuclear Engineering. Reading, MA: Addison-Wesley Publishing Co., 1975.

1. Describe contributions of Fermi, Hahn and Strassman in the discovery of fission:

2. Write a generalized nuclear equation for fission of ^{235}U:

3. Differentiate between fissionable, fissile and fertile materials:

4. List three most common fissionable nuclides:
   (1)
   (2)
   (3)

5. Write nuclear equations for converting ^{238}U into ^{239}Pu, and ^{232}Th into ^{233}U:

6. Sketch and briefly explain curve of fission cross section versus neutron energy for ^{235}U:

7. Briefly explain why most fission fragments are β^- emitters:

8. Sketch and briefly explain fission yield curve for ^{235}U:

9. Calculate energy released in fission (given identity and masses of reactants and products):

10. Briefly explain why energy of a neutrino is not usable in a power reactor:

11. Explain distribution of fission energy between three major fission products:

12. Verify that 3.1×10^{10} fissions/second produces 1 watt:

13. (Given that 280 atoms of ^{235}U undergo spontaneous fission per second per ton of material), calculate number of tons of material required to produce one watt of power from spontaneous fission:

14. Calculate power [P(t)] 10 days after shutdown if full operating power (P_o) was 100 MW; use $\frac{P(t)}{P_o} = (6.1 \times 10^{-3}) \times (t^{-0.2})$:

15. Define:

Transuranium elements.

Spontaneous fission.

Fission yield.

Unsymmetrical yield.

Decay heating.

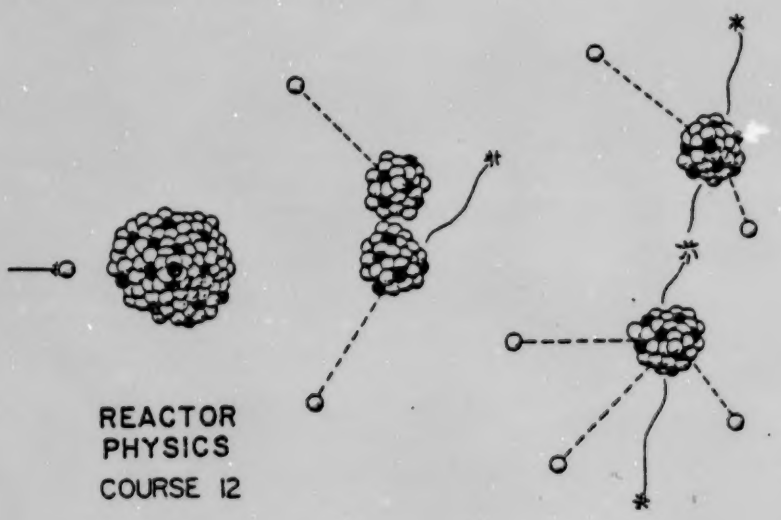
Fissile.

Fertile.

NUCLEAR TECHNOLOGY

MODULE 12-4 TIME INDEPENDENT NUCLEAR THEORY. -

RONALD L. KATHREN AND LARRY B. CHURCH



TERC-SW

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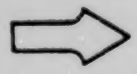
COURSE MODULES

COURSE 12 REACTOR PHYSICS

MODULE 12-1 Neutron Physics

MODULE 12-2 Cross Sections and Reactions

MODULE 12-3 Fission Rate and Reactor Power



MODULE 12-4 Time Independent Nuclear Theory

MODULE 12-5 Time Dependent Nuclear Theory

MODULE 12-6 Reactivity - Fuel Moderator and Control Systems

MODULE 12-7 Internal Effects on Reactivity

MODULE 12-8 Reactor Operating Characteristics

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INTRODUCTION

The neutron is the heart of a reactor; all reactor control is accomplished by regulation of neutrons so that a chain reaction, proceeding at the desired pace, results. This module examines the ways in which neutron interactions are controlled to most effectively sustain the chain reaction.

As the title of this module suggests, the discussion will assume that the neutron flux in the reactor cores (and hence the chain reaction) is steady-state - that is, it does not change with time.

MODULE PREREQUISITES

The student should have completed all previous modules in this Course, and their prerequisites.

OBJECTIVES

Upon completion of this module, the student should be able to:

1. Explain the concept of neutron generation.

2. Calculate the number of neutrons in a given neutron generation, when given the power level and mean neutron lifetime.
3. Give the meanings, approximate value and significance of each of the factors in the six-factor formula.
4. Define k_{eff} and k_{∞} in terms of the six factors.
5. State the relationship between k_{eff} and k_{∞}, both mathematically and physically.
6. State how resonance capture, reflectors, lack of moderation and other reactor parameters influence k_{eff} using the six-factor formula.
7. Describe how k_{eff} is calculated in a subcritical reactor.
8. Explain how the point of criticality is predicted using the subcritical multiplication technique.
9. Define the following terms:
   - Neutron generation.
   - Multiplication factor.
   - Critical.
   - Thermal neutron.
   - Fast neutron.
   - Four-factor formula.
   - Resonance region.
   - Fissile material.
   - Fissionable material.
10. Perform a horizontal and vertical flux mapping of a reactor core.
11. Perform a subcritical multiplication experiment to predict when the reactor will go critical.

NEUTRON MULTIPLICATION

The key to understanding how a reactor functions is in the concept of neutron generation. If one neutron causes a fission to occur, producing two neutrons, then the first neutron is in the first generation and the two neutrons produced by fission are in the second. If these two neutrons each cause a fission event with a total of four neutrons being formed, then these four neutrons are in the third generation. Creation of a neutron marks the beginning of a generation, and its absorption into fissile fuel to produce fission marks the end of the generation.

Clearly this is a conceptual idea rather than a physically real idea. Even if all neutrons in a generation could be created simultaneously, they would have a spread of different energies (Figure 12-4-1), travel at different velocities and interact in different ways and at different times.

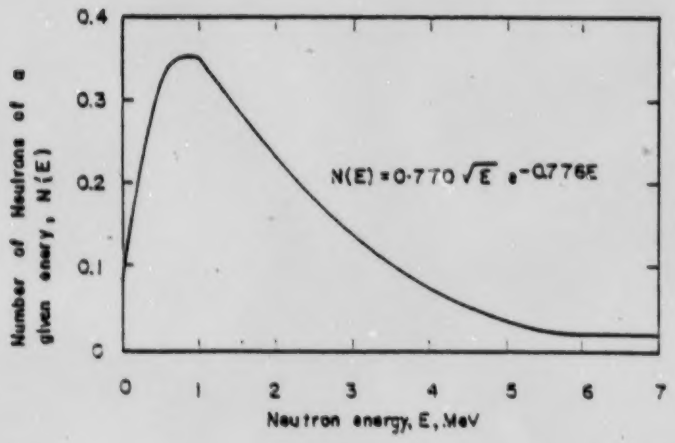

Figure 12-4-1. Energy spectrum of fission produced neutrons.

It is important to keep in mind the concept of a neutron life cycle and generation as this module is studied.

In the above example, one neutron induced a fission such that, by the start of the third generation, there were four neutrons, with the number of neutrons doubling with each generation. Thus, the number of neutrons in any succeeding generation could be determining if in fact the number were to double with each new generation. In the example cited, the multiplication factor, symbolized by $k_{effective}$ or k_{eff}, was 2.0. More precisely, k_{eff} is defined as the ratio of the number of neutrons in one generation to the number in the preceding generation, or, mathematically:

$$k_{eff} = \frac{\text{Number of fission produced neutrons in one generation}}{\text{Number of fission produced neutrons in preceding generation}}.$$

Clearly k_{eff} shows if a reactor is expanding, holding constant or decreasing its neutron flux. In the above example of $k_{eff} = 2.00$, more neutrons are produced than are being consumed. The number of neutrons (and therefore the power level of the reactor) is expanding, and the reactor is supercritical. In a supercritical reactor, $k_{eff} > 1$.

If the number of neutrons produced just equals the number used, there is no net increase or decrease in neutron flux. Hence, $k_{eff} = 1.00$ and the reactor is critical. Similarily, if the neutron flux decreases in the next generation, $k_{eff} < 1$ and the reactor is subcritical. In this case, the neutron flux will eventually be zero because multiplication as such does not occur - that is, for each neutron used to produce a fission, less than one returns, and the flux approaches (although mathematically never quite becomes) zero.

To summarize:

If $k_{eff} > 1.00$, the reactor is supercritical and the power level increases.

If $k_{eff} = 1.00$, the reactor is critical and the power level is constant.

If $k_{eff} < 1.00$, the reactor is subcritical and the power level decreases.

Note that neutron flux is directly proportional to power level.

Often a reactor core contains a neutron source that constantly emits neutrons. These neutrons are used to start the chain reaction but cannot be considered when evaluating k_{eff}. In calculating k_{eff}, only the neutrons which result from the fission of fuel can be considered.

To more fully understand the factors in a reactor that contribute to k_{eff}, often the assumption is made to eliminate all neutron leakage from the core. By not having to worry about leakage or diffussion from a core, the parameters in the core that influence k_{eff} can be more easily isolated and studied. One way of achieving zero leakage is to build an infinitely large core. In this way no neutrons can ever get to an edge of the core and leak from within its confines. Such an idea is imaginary, but it does permit theoretical study of the leakage factors which influence k_{eff}. The k_{eff} of an infinitely large core is called k_{∞}. The relationship between k_{∞} and k_{eff} is shown in Equation 12-4-1, where P is the nonleakage probability of a neutron. By definition, the P factor is ≤ 1.00.

$$\frac{k_{eff}}{k_{\infty}} = P. \qquad \text{Equation 12-4-1 (Equation = Eq.)}$$

The nonleakage factor is the fraction of neutrons that do not escape or leak out of the core, expressed by:

$$P = \frac{\text{neutrons absorbed in core}}{\text{neutrons absorbed in core + neutrons leaking from core}}.$$

This definition implies that the neutrons that do not leak from the core must remain until absorbed by the fuel, moderator or a poison.

Another way of thinking of leakage is to realize that the probability of a neutron leaking from the core is (1 - P). Clearly, in the design of a reactor it is highly desirable to increase P, thus conserving on neutrons and the required fuel that produces the neutrons. One way of accomplishing this conservation is to increase reactor size to approach the imaginary infinite reactor. There are practical limits to this approach.

It is usual for a reactor to be built with a reflector surrounding the core. The primary function of this part of the reactor is to scatter neutrons back into the core. It should therefore be constructed of a material with a high scattering cross section and a low absorption cross section. Most reflectors are also constructed of materials with low atomic masses; in this way they not only serve to reflect, but they also moderate at the same time.

Reflectors have a very strong influence on the distribution of neutrons across the core - the spatial distribution of neutrons. With no reflector, the neutron flux distribution across an unreflected or bare core would look much like that in Figure 12-4-2. If the same core were surrounded by a reflector, the neutron flux distribution would be much like that in Figure 12-4-3.

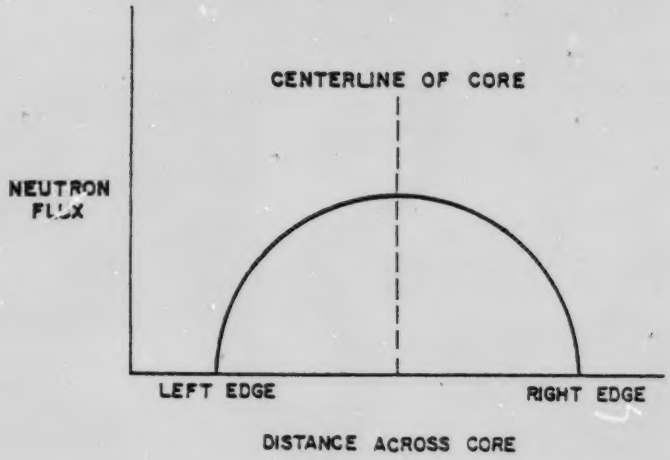

Figure 12-4-2.
Neutron flux across a symmetrical bare core.

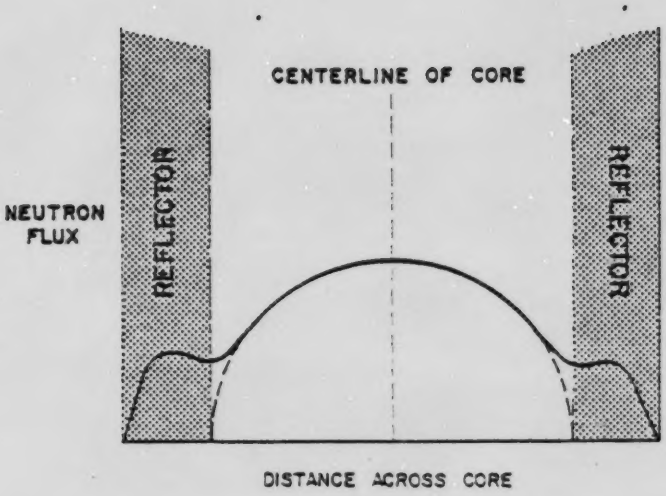

Figure 12-4-3.
Neutron flux across a symmetrical reflected core. The dashed lines show what the flux would be without the reflector.

The nonleakage probability, P, is often divided into two subgroups based upon neutron energy. Thus:

$$P = P_{th} P_f$$

where P_{th} and P_f are the nonleakage probabilities for thermal (slow) neutrons and fast (or energetic) neutrons respectively. The reason for this division is the large difference in the numerical quantities P_{th} and P_f. This can be seen in Table 12-4-1. The greater speed associated with the fast neutrons gives them a larger mean-free path and a much lower non-leakage probability when compared to thermal neutrons. Note that total nonleakage probability is very close to that of P_f, since P_{th} is very close to unity.

TABLE 12-4-1
TYPICAL NONLEAKAGE PROBABILITIES

P_f	x	P_{th}	=	P
0.64		0.93		0.60

THE FOUR-FACTOR FORMULA

In addition to the nonleakage probability, four physical events (or interactions) can occur to a neutron of any one generation. These four factors describe k_∞. Each is ex-

pressed as a probability, and when all four are multiplied together in the four-factor formula, the product is k_∞. If any one of these factors is greater than 1.00, it will contribute to increasing k_∞. Conversely, if any is less than 1.00, it will cause k_∞ to decrease. These four factors are:

Fast fission factor.
Resonance escape probability.
Thermal utilization factor.
Fuel utilization factor.

The interrelationships of these factors are shown in Figure 12-4-4; how each relates to k_∞ can be perceived by studying

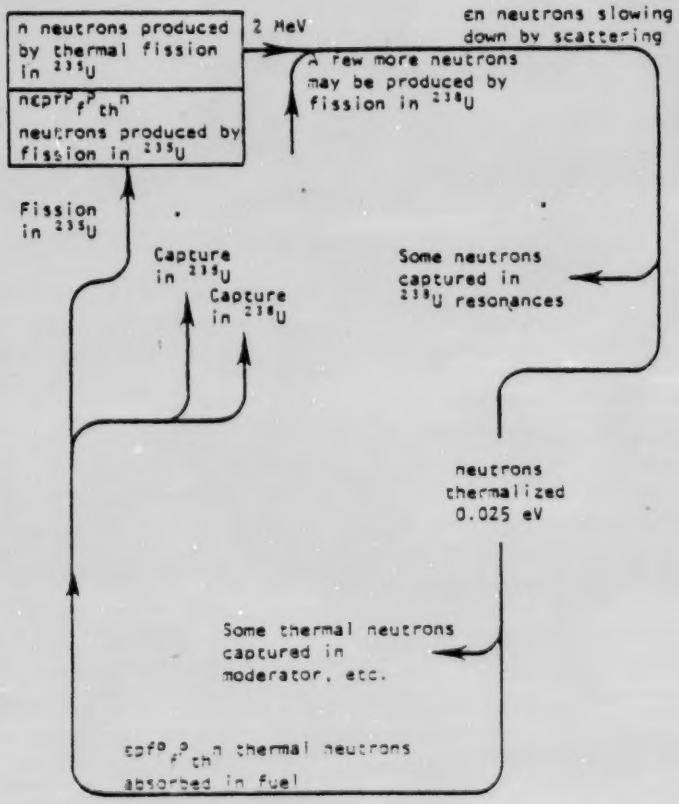
n neutrons produced by thermal fission in ^{235}U
$n\varepsilon p f^{p}_{f} {}^{p}_{th} \eta$ neutrons produced by fission in ^{235}U
2 MeV
εn neutrons slowing down by scattering
A few more neutrons may be produced by fission in ^{238}U
Fission in ^{235}U
Capture in ^{235}U
Capture in ^{238}U
Some neutrons captured in ^{238}U resonances
neutrons thermalized 0.025 eV
Some thermal neutrons captured in moderator, etc.
$\varepsilon p f^{p}_{f} {}^{p}_{th} \eta$ thermal neutrons absorbed in fuel

Figure 12-4-4. Neutron cycle in a thermal reactor. (Adapted from Bennet.)

Figure 12-4-4 as each of the four factors is discussed. The figure commences in the upper left corner by assuming that n neutrons are created at the beginning of a neutron generation.

When fission neutrons are produced, some of them have enough energy to cause fission of ^{238}U. As can be seen from the fission cross sections in Figure 12-4-5, neutrons must have an energy of greater than 1 MeV. It should be emphasized that the capture cross section of ^{238}U (which is not shown in the figure) is greater than the fission cross section. For example, at a neutron energy of 2 MeV, the capture cross section is five times as great as the fission cross section. A neutron (with 2 MeV of energy) is five times more likely to produce a ^{239}U from capture than cause fission of the ^{238}U nucleus.

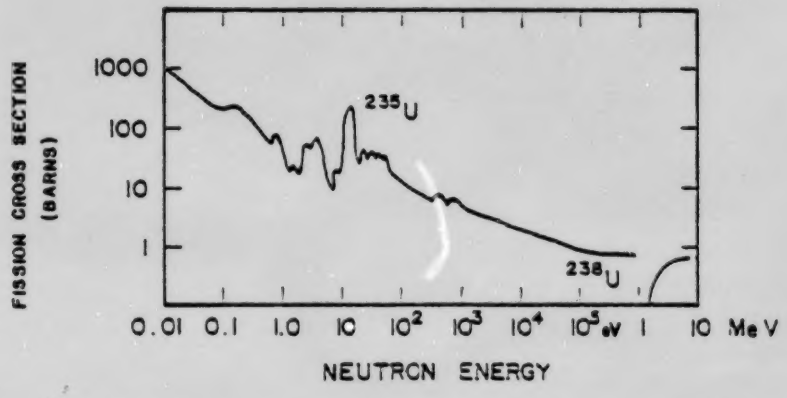

Figure 12-4-5. Fission cross section of ^{235}U and ^{238}U. (Adapted from Bennet.)

The fast fission factor, ε, is the ratio of the number of neutrons slowing down below the ^{238}U fission threshold (~ 1 MeV) to n, the number of neutrons produced per fission. After consideration of ε, there will be n ε neutrons in the

generation. The fast fission factor ($\geq$ 1.00) is a probability, dependent upon the concentration of ^{238}U in the fuel, the energy spectrum of the neutrons being created, the diameter or geometry of the fuel elements and the fuel temperature. For a typical pool reactor with 2.5 cm diameter, fuel elements are 23°C, $\varepsilon \approx 1.06$.

As neutrons leave the fuel where they were produced, they enter the moderator. Elastic collisions between the neutron and moderator slow the neutrons down from the average at which they were produced (~ 2 MeV) to thermal energies (~ 0.024 eV). As shown in Figure 12-4-6, if the neutrons pass from the moderator back into the fuel in the 5 to 500 eV region, the chances of capture by the ^{238}U in the fuel are very high due to resonances. The resonance region provides a trap for neutrons and removes them from the neutron generation. The cross sections in the resonance peaks

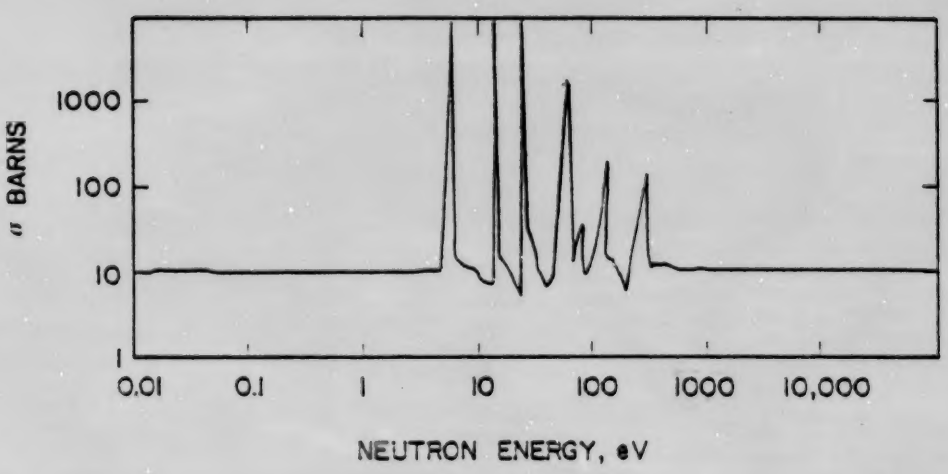

Figure 12-4-6. Total neutron cross sections for ^{238}U.

(Figure 12-4-6) for ^{238}U are of the same order of magnitude as for a typical control rod. The probability of a neutron escaping through the ^{238}U resonance region into the thermal energy region is the resonance escape probability, p. It is defined as:

$$p = \frac{\text{Number of neutrons that succeed in reaching energies lower than } ^{238}U \text{ resonance energies.}}{\text{Number of neutrons produced per fission, n}}$$

From the above discussion it should be obvious that p is ≤ 1.00 and (1 - p) is the fraction of neutrons trapped by the ^{238}U in the resonance region. The resonance escape probability is a function of the concentration of ^{238}U, the geometric shape of the fuel element, the fuel temperature and the total scattering cross section of the fuel-moderator system. Generally, p is in the 0.65 to 0.75 range.

After moderation to energies less than the resonance region there are $n \varepsilon p$ neutrons in the generation. Once in this energy region, the neutrons will continue to diffuse until they are absorbed by either the fuel, poison or moderator. Poisons are impurities in the fuel (including fission products), control rods, moderator, coolant or structural components of the reactor. A fraction of the neutrons will be absorbed by the fuel; this factor is called the thermal utilization factor, f, defined as:

$$f = \frac{\text{Thermal neutrons absorbed by the fuel}}{\text{Total thermal neutrons absorbed}}.$$

The thermal utilization factor, ≤ 1.00, depends upon the quality of fuel and moderator present in the core. In this usage, the word "fuel" means everything inside the fuel elements. Generally, f, the thermal utilization factor, varies from 0.6 to 0.8. At this point in the discussion of neutron generation, there are n ε p f neutrons present.

As mentioned, both the resonance escape probability, p, and thermal utilization factor, f, depend on how much fuel is present. Figure 12-4-7 shows the effect of the moderator-fuel ratio on these two factors, and also on their product,pf.

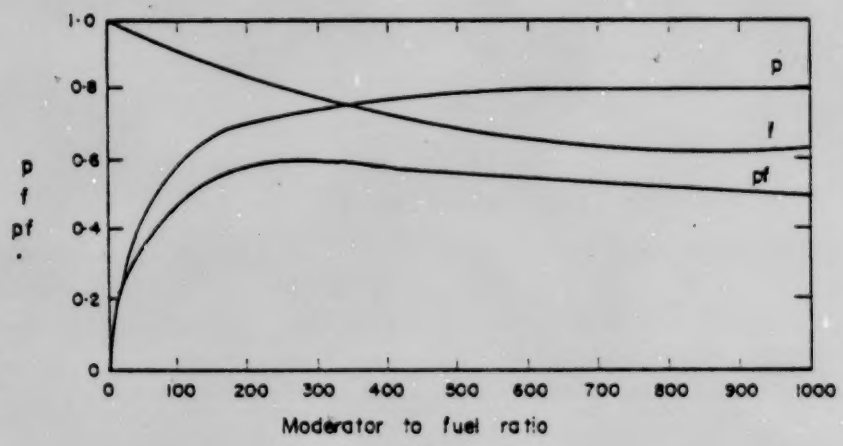

Figure 12-4-7. Variation of resonance escape probability, p, and thermal utilization factor, f, as a function of moderator-fuel ratio in a natural uranium-graphite mixture.

As the ratio approaches zero (that is, 0% moderator and 100% fuel), f approaches 1 because neutrons are absorbed only in the fuel; likewise, p approaches zero because there is no moderator to lower the neutron energies below the resonance level. Note the curve for pf, which flattens out and be-

comes essentially constant where the fuel becomes about 0.5% of the total.

Of the n ε p f thermal neutrons present in the fuel, not all will cause fission; there are a variety of competing reactions. The following thermal neutron-induced reactions can take place within the fuel:

Fission of ^{235}U.
Neutron absorption by ^{238}U.
Neutron absorption by ^{235}U.

The first of these, namely fission of ^{235}U, is the only one of importance with respect to the next neutron generation. The fuel utilization factor, η, is the number of neutrons produced by fission per neutron absorbed in the fuel, or:

$$\eta = \frac{\text{Number of fission neutrons produced}}{\text{Number of thermal neutrons absorbed by fuel}}.$$

It is important not to confuse the fuel utilization factor, η, with the number of fission neutrons produced per fission event, ν, which was defined in Module 12-3, "Fission Rate and Reactor Power." For ^{235}U, ν is 2.44 neutrons per fission event (Table 12-3-1); however, η must also take into account the neutron capture reactions of ^{235}U and ^{238}U which will reduce the number of fission-produced neutrons per neutron absorbed by the fuel. Typically, η is about 2.08 and depends upon the enrichment factor of the fuel. The exact relationship between η and ν is given by the equation:

$$\eta = \nu \frac{\text{Thermal neutrons absorbed in fission reactions}}{\text{Thermal neutrons absorbed by fuel}}.$$

Following the absorption of n ε p f neutrons by the fuel, n ε p f η neutrons are produced to start the next neutron generation. Recalling the definition of k_∞:

$$k_\infty = \frac{\text{Fission produced neutrons of one generation}}{\text{Fission produced neutrons of preceding generation}}$$

and hence

$$k_\infty = \frac{n\ \varepsilon\ p\ f\ \eta}{n}$$

or

$$k_\infty = \varepsilon\ p\ f\ \eta \qquad \text{Eq. 12-4-2}$$

Equation 12-4-2 is the four-factor formula.

THE SIX-FACTOR FORMULA

All reactors are finite in size and have some leakage of neutrons. The six-factor formula takes into account the finite size of the reactor, and accounts for this leakage by means of the nonleakage probability for both fast and thermal neutrons, or P_f and P_{th}. Thus, the six-factor formula provides k_{eff} and relates k_{eff} to k_∞ as follows:

$$k_{eff} = k_{\infty} P_f P_{th} \qquad \text{Eq. 12-4-3}$$

and, by substituting the four factors for k_{∞} in Equation 12-4-3:

$$k_{eff} = \varepsilon p f \eta P_f P_{th}$$

which is commonly known as the six-factor formula. This is the end of the cycle shown in Figure 12-4-4.

Obviously, k_{eff} will depend upon the design of the reactor and conditions within the core. Any changes to the core will change k_{eff}. Six changes to the core will be considered in terms of the six-factor formula:

Addition of pure ^{235}U.
Addition of ^{238}U.
Removal of a control rod.
Removal of moderator.
Change in reflector.
Buildup of fission products in fuel.

Addition of pure ^{235}U, or fuel, would most strongly affect the thermal utilization factor (f) since more fuel would be available for neutron absorption, in comparison to the amount of moderator and poisons. This was illustrated by Figure 12-4-7, which shows how f would increase as the ratio of moderator to fuel decreased. The fuel utilization factor, η, is also strongly influenced because the percentage of ^{238}U is reduced, which decreases the thermal neutron capture by ^{238}U relative to fission of ^{235}U. Undoubtedly, the overall

effect of ^{235}U addition to a reactor core would be to increase k_{eff}, and conversely the removal of ^{235}U would decrease k_{eff}. The same also holds true for enrichment and for the same reasons: a core of highly enriched uranium (say 40%) will have a much higher k_{eff} than a core of natural uranium, which has 0.7% of ^{235}U.

An addition of ^{238}U will influence almost every factor of the four-factor formula. The fast fission factor will be increased due to the increase in ^{238}U fission. The resonance escape probability, p, will probably be the most strongly influenced, and will drop in direct proportion to the concentration of ^{238}U in the core. The thermal utilization factor will increase if ^{238}U is added, but the fuel utilization factor will decrease. This is because η is proportional to the ratio of thermal neutrons absorbed in fission to thermal neutrons absorbed by fuel for other reactions. Since there is no change in the amount of ^{235}U, there is no change in the number of neutrons absorbed in thermal fission reactions. There is, however, more ^{238}U in the fuel, and this increases the number of thermal neutrons absorbed in other reactions. The overall effect is a decrease in k_{eff}.

The removal of a control rod will most strongly influence the thermal utilization factor (f) because as the control rod or poison is withdrawn, more neutrons will be available for absorption into the fuel. With more neutrons entering the fuel, more fissions can occur and hence k_{eff} is increased.

The removal of moderator will most strongly affect the resonance escape probability, p, and the fast neutron nonleakage probability P_f. With less moderator in the core, the energy spectrum of the neutron will be shifted to higher energies. Figure 12-4-5 illustrated what happens to p as

the moderator concentration drops: a large fraction of the epithermal neutrons are captured by the ^{238}U and p decreases. Likewise, with less moderator, these same fast neutrons have a better chance of escaping from the core before being thermalized. Some nuclear scientists would disagree with the conclusion concerning the effect of moderator upon the fast nonleakage probability. Their argument would be that, in considering the nonleakage probability, one must consider the ratio of fast neutrons not leaking to the total fast neutron population; in other words, something such as the moderator which changes the neutron energy-spectrum cannot be considered in a contemplation of the nonleakage probability. It is left to the reader to decide which side of this question is to be supported. In any case, all will agree that with less moderator k_{eff} is lowered.

A change in reflector will primarily affect P_f and P_{th}, the nonleakage probabilities. Addition of reflector will increase the probability of nonleakage, particularly for thermal neutrons. More neutrons, and in particular thermal neutrons, will be available and k_{eff} will increase. Conversely, removing reflection will lower k_{eff}.

Buildup of fission products in fuel will be discussed more fully in a later module in this Course, but should be mentioned briefly. Many of these fission products have high capture cross sections for thermal neutrons and compete effectively with ^{235}U for thermal neutrons. If the fission products are considered part of the fuel, then they will primarily affect f, the thermal utilization factor. They will also affect the fuel utilization factor, η, since the number of thermal neutrons absorbed by the fuel will be increased, but the number producing fission will be reduced because of the competitive reactions. Fission product build-

up will reduce k_{eff}, and fresh fuel is always more reactive (higher k_{eff}) than fuel containing fission products.

SUBCRITICAL MULTIPLICATION

Subcritical multiplication (when $k_{eff} \leq 1.00$) is standard operating procedure. When loading a new reactor with fuel, a subcritical multiplication experiment is performed to ascertain the amount of fuel necessary to go critical. The experiment is a classic in nuclear science; the experiment is found at the end of this module and the theory is presented now.

If an external neutron source is present in a subcritical assembly of ^{235}U, fissions will occur in the fuel and a steady-state flux will be reached. If changes are made in either the amount of fuel, its configuration or the position of the control rods, there will be a change in this steady-state flux. If it is assumed that the flux increases but the reactor remains subcritical, then the increase is referred to as subcritical multiplication. The subcritical multiplication factor, M, is the ratio of the total number of neutrons from the source and fuel, divided by the neutrons from the source alone:

$$M = \frac{\text{Total number of neutrons (source and fuel)}}{\text{Number of neutrons from source only}}.$$

If the source alone yields a number (n) of thermal neutrons, the first generation of fission-produced neutrons will be nk_{eff}. This number of neutrons will be symbolized by n_1. Similarily the second generation fission neutrons will be

$n_1 k_{eff} = n_2 = n(k_{eff})^2$. After a number, say m, generations, a steady-state level of neutrons will be reached, and the total number of neutrons at any one time is given by the following equation:

$$n_{total} = n + n_1 + n_2 + n_3 + \ldots n_m$$

$$n_{total} = n + nk_{eff} + nk_{eff}^2 + nk_{eff}^3 + \ldots nk_{eff}^m.$$

Eq. 12-4-4

The subcritical multiplication factor, M, is therefore obtained by dividing Equation 12-4-4 by n. and is:

$$M = 1 + k_{eff} + k_{eff}^2 + k_{eff}^3 + \ldots k_{eff}^m. \qquad \text{Eq. 12-4-5}$$

Equation 12-4-5 can be solved mathematically to yield:

$$M = \frac{1}{1-k_{eff}}. \qquad \text{Eq. 12-4-6}$$

As the reactor approaches criticality, k_{eff} approaches 1.00 and M approaches infinity. The exact value for k_{eff} can be determined after any subcritical change by measuring M and solving Equation 12-4-6 for k_{eff}. The point at which $k_{eff} = 1.00$ and the reactor going critical can be determined by taking the inverse of Equation 12-4-6:

$$\frac{1}{M} = 1 - k_{eff}.$$

As will be seen in the experiment, the amount of the fuel or position of the control rods at which the reactor will become critical is determined by plotting some parameter of core configuration - such as control position or number of fuel elements loaded - against 1/M. When the curve of 1/M versus fuel mass is extrapolated to zero, the quantity of fuel required to make the reactor exactly critical (i.e., $k_{eff} = 1.000$), can be estimated at each step of the loading process.

CRITICAL MASS

An important concept is that of critical mass, or how much fissile material (i.e., ^{235}U) is needed to support a chain reaction or achieve criticality. The amount is dependent upon many variables, including enrichment, form (e.g., metal versus oxide versus solution), moderation, reflection and geometry. The latter, geometry, is a major consideration, for a mass of fuel in one particular configuration might not be critical. A quantity of fuel greater than a critical mass can spontaneously go supercritical using neutrons it generates internally or a stray neutron in space to start the multiplication process. This is known as a nuclear excursion or criticality accident.

The smallest mass that would achieve criticality would have a small surface-to-mass ratio such that relatively few neutrons would escape. The geometric figure with the smallest surface-to-mass ratio is a sphere. A few kilograms of

highly enriched ^{235}U or pure ^{239}Pu metal in the shape of a sphere could go critical on its own, while the same mass, if in the shape of a slab a few centimeters thick, would be subcritical.

Similarily, a sphere of highly enriched ^{235}U metal would be subcritical in air, but if dropped into a bucket of water, might spontaneously go critical because of the moderation afforded by the water. If the fissile material is put into water, moderation is enhanced, and yet a smaller mass would be required for criticality. Under ideal conditions, a solution, totally moderated and reflected and in spherical shape - could achieve criticality with less than a pound of fissile material. Fissile refers to materials capable of achieving a self-sustaining chain reaction or achieving criticality (the three reactor fuels: ^{235}U, ^{238}U and ^{239}Pu). Fissionable materials can be split by neutrons, but the value of k_{eff} cannot be made to equal or exceed 1, usually because of low fission cross sections and too few neutrons emitted per fission.

EXERCISES

1. Explain the difference between the four-factor formula and the six-factor formula.
2. Describe the effect of each of the following upon k_{eff}. Answer in terms of the six-factor formula.
   a. The addition of ^{238}U to the core.

b. The removal of all reactor water (neglect all heating effects).

c. The addition of a large amount of Na to the reactor water.

3. If there are 25 neutrons in the first generation and k_{eff} is 1.050, how many neutrons will be present in the fifth generation?

4. Explain the difference between k_{eff} and k_{∞}.

5. Define the terms listed in point 8 of the Objectives.

MATERIALS

A small experimental or research critical reactor (although a subcritical assembly will suffice).

A pure metal such as Au, Co, Cu or Mo.

A counting device capable of counting the neutron activation product of the above metal (for example, a β-counter or NaI detector with the necessary counter).

Graph paper, linear Cartesian.

A low count rate neutron chamber output from the reactor.

1. HORIZONTAL FLUX MAP OF A REACTOR CORE

Cut out and accurately weigh small pieces of the activation foil to be used in this experiment. The instructor will advise you on the approximate quantity for each sample. It will probably be desirable to make those pieces destined for the outer regions of the core larger than those for the interior parts of the core. Record the weight data on Data Table 12-4-1.

Place each sample into the reactor core to cover one axis from the center of the core to outside the reflector. Be sure that all samples are exactly at the same vertical position (i.e., they should all be in the same horizontal plane). Note the positions on Data Table 12-4-1; if necessary, consult the reactor blueprints to determine the distance from core center.

Irradiate the samples, noting on Data Table 12-4-2 the details of the irradiation.

When advised by your instructor, remove the samples from the reactor and begin to count. (Before counting, look up in an appropriate reference book, such as the CRC Handbook of Radionuclides or Radiological Health Handbook, the decay scheme of your activated product.) Count each sample at least once so that a minimum of 10^4 counts are recorded. Enter the count data into Data Table 12-4-3.

Correct each count for the decay which has occurred since the end of the irradiation using $A_o = A_t e^{\lambda t}$. If possible, correct for the counter efficiency, ε, and the decay data for the nuclide. (This last factor is often called branching ratio, BR.) Convert A_o to D_o, the dis-

integration rate at the end of irradiation using:

$$D_o = \frac{A_o}{\varepsilon(BR)}.$$

If only calculation of A_o is possible, plot on linear graph paper A_o/mass against distance from the center of the core for each sample. The value of A_o/mass is proportional to the neutron flux and is called the relative flux.

If D_o can be obtained, it may be instructive to plot distance as a function of neutron flux, ϕ, also called the absolute flux. The latter may be calculated from D_o, using Equation 12-4-7 where N is the number of atoms in each sample, irradiated to produce the activated product which was counted, σ is the cross section of the activation reaction and T is the length of the irradiation. In solving for the flux, be sure that λ and t have the same reciprocal units.

$$\phi = \frac{D_o}{N\,\sigma\,(1-e^{-\lambda t})}. \qquad \text{Eq. 12-4-7}$$

2. VERTICAL FLUX DISTRIBUTION MAP OF A REACTOR CORE

In a manner very similar to the first experiment, cut out small pieces of target metal which will be placed in a vertical column to measure the flux. In positioning the samples, an arbitrary reference point will have to be selected and all foils placed a known distance above or below this point. Using Data Tables

12-4-4, 12-4-5 and 12-4-6, plot the relative flux (A_o/mass) or absolute flux data, as a function of the vertical distance from the reference point for each sample.

3. SUBCRITICAL MULTIPLICATIONS

The purpose of this experiment is to predict when a reactor will go critical using the approach outlined in the test.

It will be assumed that the reactor is already fueled and the approach to criticality will be made by the withdrawal of a control rod. If such is not the case, the experiment will have to be modified slightly to achieve the same goals. When a reactor is fully fueled with a source in place and is subcritical because of the control rods' positions, it is impossible to get a value for n alone. This is because there already exists a certain level of subcritical multiplication occurring within the core at all times (i.e., k_{eff} is not exactly 0.00 when the experiment begins as was tacitly assumed in the theory writeup). It will be assumed that the initial reading of the count rate on the neutron chamber is very close to that from the source alone.

After any safety control rods have been removed from the core, take a reading of the count rate on the low count rate neutron chamber. This will represent n. Remove the control rods until there is a noticeable change in the count rate. After waiting several minutes for the count rate to stabilize at the new level, record the new value in Data Table 12-4-7. Calculate M, the subcritical multiplication factor, using the equation below, and then 1/M:

$$M = \frac{\text{Count rate with control rod up}}{n}.$$

Plot 1/M as a function of control rod position, assuming 1/M = 1.00 with the rod full in. Make a crude estimate as to what control rod position will be necessary to achieve criticality: 1/M = 0.00. Move the control rod approximately half the distance between the present position and the estimated criticality position and repeat the reading. Make another estimate of criticality and continue the procedure of moving it half way to the position where k_{eff} will equal 1.00.

When the control rod is about 95% of the total estimated distance to criticality, remove the source. If the reactor is still subcritical, the power or count rate should decrease. Replace the source and move the control to the estimated k_{eff} = 1.00 position and again remove the source. If the reactor is critical, the count rate should not decrease.

DATA TABLE 12-4-1
HORIZONTAL FLUX DISTRIBUTION
SAMPLE MASS AND POSITIONS

No.	Mass	Distance from Core Center, cm
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
.		
.		
.		

DATA TABLE 12-4-2
HORIZONTAL FLUX DISTRIBUTION
IRRADIATION DATA

Irradiation date: _____

Power level: _____

Start time: _____

Stop time: _____ = End of irradiation

Net irradiation time: _____

Target metal used: _____

Radionuclide counted: _____

Half-life: _____

λ: _____

Branching ratio: _____

Counter efficiency: _____

Background count rate: _____

DATA TABLE 12-4-3
HORIZONTAL FLUX DISTRIBUTION
COUNTING DATA

Sample No.	Count Start Time	A, Net Count Rate	$A_o = Ae^{\lambda t}$	Relative Flux A_o/Mass	Absolute Flux ϕ
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					

DISCUSSION:

Discuss the A_o/Mass (relative flux) or ϕ (absolute flux) curves. Does the shape look like that in Figure 12-4-2 or 12-4-3? Explain any differences.

DATA TABLE 12-4-4

VERTICAL FLUX DISTRIBUTION

SAMPLE MASS AND POSITIONS

No.	Mass	Distance from Reference Point
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
.		
.		
.		

DATA TABLE 12-4-5
VERTICAL FLUX DISTRIBUTION
IRRADIATION DATA

Irradiation date: _____

Power level: _____

Start time: _____

Stop time: _____ = End of irradiation

Net irradiation time: _____

Target metal used: _____

Radionuclide counted: _____

Half-life: _____

λ: _____

Branching ratio: _____

Counter efficiency: _____

Background count rate: _____

DATA TABLE 12-4-6
VERTICAL FLUX DISTRIBUTION
COUNTING DATA

Sample No.	Count Start Time	A = Net Count Rate*	$A_o = Ae^{\lambda t}$	Relative Flux A_o/Mass	Absolute Flux ϕ
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					

DISCUSSION:

Discuss the shape of the relative or absolute flux curves. Explain any unusual shapes.

DATA TABLE 12-4-7
SUBCRITICAL MULTIPLICATION

No.	Rod Position	Count Rate	M	1/M	Predicted Position of Criticality
1	Full in	= n	---	---	---
2					
3					
4					
5					
6					
7					
8					
9					
10					
.					
.					
.					

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Glasstone, Samuel, and Sesonske, Alexander. Nuclear Reactor Engineering. New York: D. Van Nostrand, 1967.

Grant, P. J. Elementary Reactor Physics. Oxford: Pergamon Press, 1966.

LaMarsh, John R. Introduction to Nuclear Engineering, Reading, MA: Addison-Wesley Publishing Co., 1975.

Murray, Raymond L. Nuclear Energy. New York: Pergamon Press, 1975.

Pedersen, Knud B. et al. Applied Nuclear Power Engineering for Practicing Engineers. Boston: Cahners Books, 1972.

1. Briefly explain concept of neutron generation (in terms given in this module):

2. Given a multiplication factor (k_{eff}) of 0.8 and information that one neutron causes a fission to occur, calculate preceding generation of neutrons:

3. Use six-factor formula (given $k_{eff} = \varepsilon\ p\ f\ \eta\ P_f\ P_{th}$); write in table meaning and appropriate value of each formula element:

4. Define k_{eff} and k_{∞} in terms of six factors:

5. Explain:

   a. Mathematical relationship between k_{eff} and k_{∞}:

b. How this relationship is expressed in physical terms:

6. Listed below are three reactor parameters; describe how each influences multiplication factor (k_{eff}) (use six-factor formula):

   a. Resonance capture.

   b. Reflectors.

   c. Lack of moderation.

7. Explain how k_{eff} is calculated in a subcritical reactor:

8. Define (with terms used in this module):

Multiplication factor.

Neutron generation.

Critical.

Thermal neutron.

Fast neutron.

Four-factor formula.

Resonance region.

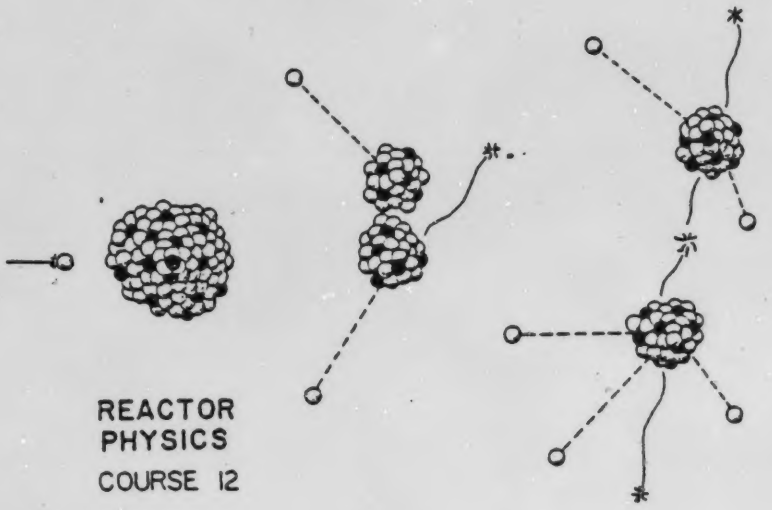
Fissile material.

Fissionable material.

NUCLEAR TECHNOLOGY

MODULE 12-5 TIME DEPENDENT NUCLEAR THEORY. -

RONALD L. KATHREN AND LARRY B. CHURCH



TERC-SW

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COURSE MODULES

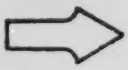
COURSE 12 REACTOR PHYSICS

MODULE 12-1 Neutron Physics

MODULE 12-2 Cross Sections and Reactions

MODULE 12-3 Fission Rate and Reactor Power

MODULE 12-4 Time Independent Nuclear Theory



MODULE 12-5 Time Dependent Nuclear Theory

MODULE 12-6 Reactivity - Fuel Moderator and Control Systems

MODULE 12-7 Internal Effects on Reactivity

MODULE 12-8 Reactor Operating Characteristics

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INTRODUCTION

In the previous module, the reactor was considered as if its operation were independent of time - a situation which does not exist in the real world. The reactor is very dependent on time or, more specifically, is controlled by processes that are time dependent. Reactor kinetics refer to behavorial changes in reactor power levels with time. Kinetics, as used in this sense, refers to time dependent behavior (and should not be confused with kinetic, which refers to motion).

In this module, time dependent behavior of a reactor is examined.

MODULE PREREQUISITES

The student should have completed the previous module (12-4) in this Course, along with its prerequisites.

OBJECTIVES

Upon completion of this module, the student should be able to:

1. Define reactor period.
2. Give a typical value for a prompt neutron generation lifetime.

3. Explain why a reactor is uncontrollable by ordinary methods when operated only on prompt neutrons.
4. Explain, by example, effect of delayed neutrons upon reactor behavior.
5. List typical values of k_{eff} and β for subcritical, critical and supercritical reactors.
6. Explain concept of reactivity.
7. Given a reactivity-versus-period graph, find reactivity corresponding to any specified period.
8. Define:
   - Δk (excess multiplication factor).
   - Prompt neutron lifetime (ℓ_p).
   - Delayed neutrons.
   - β.
   - In-hour equation.
   - Reactor period.
   - Prompt critical.
   - In-hour curve.
9. Measure decay rate of delayed neutrons following a sudden shutdown (compare with theory).

REACTOR KINETICS: PROMPT NEUTRONS

In a reactor operating in a steady state, (usually) there is no change with time; thus, if steady state operation were all that was expected of a reactor, there would be no need to evaluate and study kinetics; however, any change in steady state (startup, shutdown or power changes of any magnitude) has a certain time dependence, and this time dependence is the basis of reactor control.

Reactivity is the term used to describe the departure of a reactor from criticality. The effective multiplication factor, k_{eff}, determines whether the neutron density within a reactor will remain constant or change. Since the power level is directly proportional to neutron density, whenever $k_{eff} = 1.000$, the reactor is critical and operates at a constant power level. If k_{eff} is less than 1.000, the reactor is subcritical and the power level decreases. Conversely, if k_{eff} is greater than 1.000, the reactor is supercritical and the power level rises. The power level and neutron density are constantly changing whenever k_{eff} is not equal to 1.000.

The difference between the value of k_{eff} and 1.000 is known as the excess multiplication factor, Δk, which is defined in Equation 12-5-1:

$$\Delta k = k_{eff} - 1.000. \qquad \text{Equation 12-5-1}$$

(Equation = Eq.)

The excess multiplication factor, Δk, may be either positive or negative, depending upon whether k_{eff} is greater or smaller than 1.000.

Reactivity, ρ, is the ratio of the excess multiplication factor, Δk, to the effective multiplication factor, k_{eff}, and can be expressed mathematically as:

$$\rho = \frac{\Delta k}{k_{eff}}$$

$$\text{or } \rho = \frac{k_{eff} - 1.000}{k_{eff}}. \qquad \text{Eq. 12-5-2}$$

Whenever the reactor is just critical, $k_{eff} = 1.000$. In this situation, ρ and Δk are equal. When k_{eff} is slightly smaller or larger than 1, ρ and Δk are almost identical.

The reactivity, ρ, can be either positive or negative. When $k_{eff} > 1.000$, ρ is positive, and the reactor has positive reactivity. In this situation, ρ will have a value greater than 0 and less than 1 (verified by Equation 12-5-2). When $k_{eff} < 1.000$, ρ is negative, and the reactor has a negative reactivity. In this situation, ρ can have any value < 0, or mathematically, $-\infty < \rho < 0$.

Reactivity is a very useful concept. The reactivity, or effect on multiplication, of any material can be determined. It is often used to describe the effect of materials that might be inserted into the reactor, or the effectiveness of a control rod. Thus, were a piece of pure ^{235}U to be inserted into the core, it would enhance the reaction because it has a positive effect on reactivity. (It is said to have positive reactivity.) The control rod absorbs neutrons and reduces multiplication; it has a negative reactivity.

An important quantity in reactor kinetics is prompt neutron lifetime (ℓ_p) - the average time elapsed between the production

of a neutron from a fission reaction and its loss from the free neutron pool by either escape or absorption. In considering the life cycle of a neutron, it is quickly recognized that ℓ_p is the sum of two separate parts: the slowing down time, which is the average time required for the fast neutron to slow to thermal energies, and the diffusion time, which is the average time that the thermal neutron diffuses before it either escapes or is captured.

In a light water moderated reactor, the diffusion time is about 2×10^{-4} seconds; the slowing down time is around 7×10^{-6} seconds. Hence:

$$\ell_p = (2 \times 10^{-4}) + (7 \times 10^{-6}) \text{ seconds} \qquad \text{Eq. 12-5-3}$$

$$\ell_p \simeq 2 \times 10^{-4} \text{ seconds.}$$

Table 12-5-1 gives the slowing down and diffusion times for neutrons of four different moderators.

TABLE 12-5-1
SLOWING DOWN AND DIFFUSION TIMES IN AN INFINITE MEDIUM

Moderator	Slowing Down Time	Diffusion Time
Light water	7.1×10^{-6} s	2.4×10^{-4}
Heavy water	5.0×10^{-5}	6.0×10^{-2}
Beryllium	5.7×10^{-5}	4.2×10^{-2}
Graphite	1.4×10^{-4}	1.6×10^{-2}

Most of the neutrons produced in the fission process are prompt neutrons - neutrons produced at the instant of fission. The time-dependent behavior of the reactor, at first made critical with only prompt neutrons, is then made supercritical by a sudden increase in k_{eff}.

If normal reaction time of a typical reactor operator is only two seconds, the reactor may increase in power by a factor of more than 22,000 by the time an operator can act. This very rapid increase in power level makes the reactor extremely difficult if not completely impossible to control.

The discussion from the preceding paragraph shows that in the absence of something which lengthens the average neutron lifetime, the reactor power levels will change very rapidly as a result of small reactivity changes; and, control during even a tiny increase in power would be most difficult, if not impossible.

REACTOR KINETICS: DELAYED NEUTRONS

As suggested in the above example, the solution to the problem of safely controlling a reactor is to be able to prolong the lifetime of the neutrons in the reactor; this is accomplished by taking advantage of the existance of delayed neutrons. Prompt neutrons constitute more than 99% of the total fission neutrons and are released within 10^{-14} seconds of the instant of fission. Delayed neutrons are emitted as a result of the decay of fission products; they are not released from the primary fission process. The delayed neutrons are emitted over a period of a few seconds following the fission event. Because the delayed neutrons come from the decay of the fission products, they are emitted with a characteristic set of half-lives ranging from 0.2 seconds to 55 seconds (Table 12-5-2). Figure 12-5-1 represents the decay scheme of the A = 87 chain.

TABLE 12-5-2

DELAYED NEUTRON DATA FOR THE THERMAL FISSION OF ^{235}U.

Group	Half-Life Sec	Decay Constant λ_1, Sec^{-1}	Energy keV	Mean Life, T (seconds)	Fraction β_1
1	55.72	0.0124	250	80.2	0.000215
2	22.72	0.0305	560	32.7	0.001424
3	6.22	0.111	405	8.9	0.001274
4	2.30	0.301	450	3.3	0.002568
5	0.610	1.14	-	0.88	0.000748
6	0.230	3.01	-	0.33	0.000273

Total yield: 0.0158 neutrons per fission.
Total delayed fraction (β): 0.0065.

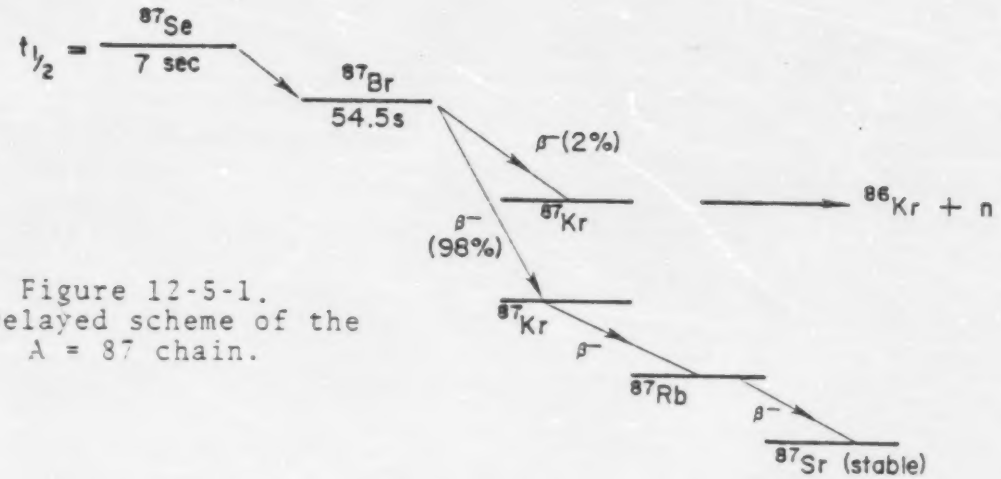


Figure 12-5-1.
Delayed scheme of the
A = 87 chain.

The fission product is ^{87}Se, which decays by β^- emission to ^{87}Br with a half-life of 7 seconds. The ^{87}Br with a half-life of 54.5 seconds, in turn decays by β emission to ^{87}Kr. Most of the ^{87}Kr so formed is in a low energy state, but 2% is in such an excited state that it decays immediately by neutron emmission to form ^{86}Kr. It is therefore a source, although a very small one, of neutrons whose emission is delayed by an average of at least 80 seconds (the mean life of ^{87}Br) after the fission event. In the case of ^{235}U, the total fraction of neutrons delayed is only about 0.65% of the total neutron yield. However, they play a vital role in the control of the nuclear reactor.

Experimental studies of the rate of emission of delayed neutrons have shown that all of the delayed neutrons fall into one of six groups, and each is characterized by a definite half-life. The average time the emission of each group of neutrons is delayed is equal to the mean life ($t_{\frac{1}{2}}/0.693$) of the corresponding delayed neutron precursor. The average number of delayed neutrons per fission in each group and the total number of fission neutrons that are delayed are presented in Table 12-5-2 for the total fission of uranium-235. It should be noted that the approximate energies for the delayed neutrons are lower than for the great majority of prompt neutrons.

The total fraction of neutrons which are delayed is called β. As can be determined by the addition of the last column of Table 12-5-2, β for uranium-235 is equal to 0.0065 or 0.65%.

The primary influence of delayed neutrons is to provide a more reasonable or longer period as the reactor power is increased. However, a less obvious effect of delayed neutrons can be observed as the reactor is increased from one power level to another. Assuming that this increase in power level is relatively fast compared to the 55 second, longest-lived, delayed-neutron precursor, the reactor will arrive at the higher

power level with the instantaneous effective β being less than 0.65%. This is because the full complement of delayed neutrons has not yet been created. If the reactor is immediately made just critical at the higher power level, it will be observed that, as more delayed neutrons are formed, the power level will continue to increase unless corrective action is taken. The normal action is to slowly drive the control rods in until all of the delayed neutrons are in an equilibrium state and β is exactly equal to 0.65%.

The exact length of time during which the control rods must be driven in after attaining a given power level depends on the speed with which the power level increase is made. If it was fast, then few delayed neutrons of the longer-lived precursors will be in equilibrium and the length of time required to attain equilibrium will be at least five minutes. If the power level increase took place over a relatively long period of time (for example, several hours) then the higher power level will be reached with almost the full complement of delayed neutrons present. The control rods will not need to be driven in very much, if at all.

When the reactor power level is dropped suddenly, a set of circumstances exactly opposite to those described for a power increase will occur. Upon reaching the lower power level, the fraction of delayed neutrons will be in excess of β. This is because the delayed neutrons produced by the fission events at the higher power level have not yet had a chance to decay. As they decay, the fraction of neutrons which are delayed will approach β and the neutron flux will diminish. The control rods will have to be withdrawn at the lower power level to compensate for the diminishing neutron density. The extent of time over which they will have to be withdrawn is a function of how quickly the power decrease was made. An

extreme example of a power decrease is given in the second Procedure of this module, where the control rods are placed completely and suddenly into the reactor core. Following such an action, the primary source of neutrons is the delayed neutrons.

There is a difference between the theoretical β (0.0065 for uranium-235) and the real, or effective, value for β observed in a working reactor, often called β_{eff}. The difference between β and β_{eff} is due to the difference of the neutron energy distribution of prompt and delayed neutrons. From observation of Table 12-5-2, delayed neutrons have, on the whole, a lower energy distribution than prompt neutrons. As a result, they are thermalized more quickly and will have less opportunity to escape from the core before being thermalized or absorbed. In other words, more prompt neutrons escape from the core than delayed neutrons. As a result, the total fraction of delayed neutrons in the core is very often greater than the calculated β. In a typical research reactor, fueled with ^{235}U, β_{eff} equals 0.0075.

THE IN-HOUR EQUATION

By taking into account the six groups of delayed neutrons, Δk of a reactor can be related to the prompt neutron lifetime, ℓ_p; the reactor period (due to all neutrons in the core); T; β; and T_i for each group of delayed neutrons. Equation 12-5-4 may be simplified by dropping the summation sign and assuming a weighted average value for the various T_i, labeled $\overline{T}$:

$$\Delta k = \frac{\ell_p}{T} + \sum_{i=1}^{6} \left(\frac{\beta_i T_i}{T + T_i} \right). \qquad \text{Eq. 12-5-4}$$

The value of T_i is 12.3 seconds. Since the sum of β_i is equal to β, Equation 12-5-4 may be replaced by the approximation of Equation 12-5-5:

$$\Delta k = \frac{\ell_p}{T} + \frac{\beta \bar{T}}{T + \bar{T}}. \qquad \text{Eq. 12-5-5}$$

Equation 12-5-5 is a form of the in-hour equation, which, in its purest sense, is used to calculate the reactivity in units of inverse or reciprocal hours. The in-hour unit is defined as the amount of reactivity which would make the stable reactor period equal to one hour. (Reactor period is the length of time needed to change the reactor power, or neutron flux, by a factor of e.) If this equation is solved for T, the solution is quadratic. The larger of the two positive roots, known as the dominant root, can be approximated by Equation 12-5-6:

$$T \approx \frac{\ell_p + (\beta - \Delta k)\ T}{\Delta k}. \qquad \text{Eq. 12-5-6}$$

Equation 12-5-6 is used routinely to relate the excess multiplication factor, Δk, to the period. In Equation 12-5-6, the term ℓ_p is the contribution of prompt neutrons to the period while the term $[(\beta + \Delta k)\ T]$ is the delayed neutron contribution.

By returning to equation 12-5-2:

$$\rho = \frac{\Delta k}{k_{eff}}.$$

Note that as $k_{eff} \approx 1.000$, ρ is approximately equal to β. Thus, when $\rho \approx \beta$ [that is, when $(\beta - \Delta k) \approx 0$], the stable period as calculated by Equation 12-5-6 approaches the period due to prompt neutrons only. When enough reactivity has been placed in a reactor such that $(\beta - \Delta k) = 0$, the reactor is known as prompt critical - in other words, it is critical on prompt neutrons alone. The period will be very short, and a potentially dangerous situation may be created since there may be insufficient time to control the reactor.

Such a situation could occur from a sudden increase in reactivity. Suppose a control rod with a reactivity worth of $\Delta k = 0.0065$ were suddenly removed from a just critical reactor. In this example, the removal of the control rod is equivalent to the addition of $\Delta k = 0.0065$, a value chosen because it exactly equals β.

The relationship between reactivity and stable period is shown in Figure 12-5-2 for a ^{235}U-fueled reactor. This is

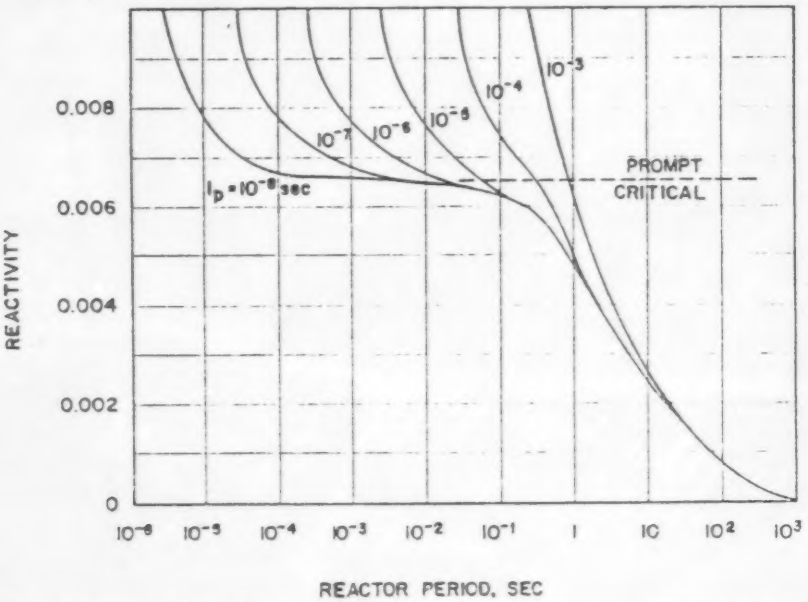


Figure 12-5-2. Relationship between reactor period, T, and reactivity, ρ, for various prompt neutron lifetimes. (Glasstone & Sesonske, page 245.)

sometimes known as the in-hour curve, and provides a good picture of the effects of prompt neutron lifetime. Note that, as the period increases above about 10 seconds, the prompt neutron lifetime becomes unimportant insofar as reactivity is concerned. However, once the reactivity exceeds 0.0065 (prompt critical), the resultant period is a strong function of the prompt neutron lifetime. At low reactivities, all the curves coincide and for a specified reactivity, the period is independent of the prompt neutron lifetime.

By substituting Equation 12-5-6 into an exponential equation and considering a sudden increase in reactivity, it is found that, for a very short time after the reactivity insertion, the reactor behaves as if all the additional neutrons were prompt. There is a sudden steep change in the neutron flux (called the transient period) followed by a steady constant rate of change (called the stable reactor period) which is approximated by Equation 12-5-6. A plot of n/n_o as a function of time is shown in Figure 12-5-3; during the first second or so, the transient period predominates; this is followed by the stable period as seen by the line with constant slope line for the remainder of the curve.

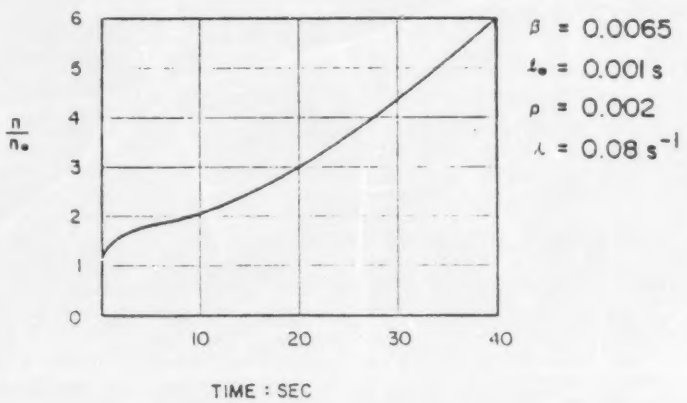


Figure 12-5-3. Response to a reactor to a positive step change of reactivity.

Because of its special significance, the prompt critical condition is sometimes used as the basis for a reactivity unit. The unit of reactivity is called the dollar and is defined in Equation 12-5-7.

$$\text{Reactivity in dollars} = \frac{\rho}{\beta_{eff}} = \frac{\Delta k}{\beta k_{eff}}. \qquad \text{Eq. 12-5-7}$$

When prompt critical, a reactor has a reactivity of exactly one dollar. The insertion of \$0.10 (10 cents) to a critical reactor implies that the reactor is brought one-tenth of the way between being critical and prompt critical.

The value of the dollar was chosen because it is related not only to reactivity, ρ, but also to β. Table 12-5-3 shows that β is a function of the fuel. Thus, a given value of ρ may produce a wide variety of responses within the reactor. The use of the dollar unit eliminates the problem of solving Equation 12-5-5 for T, since there is also a negative root which will apply when Δk is negative. In such a situation,

TABLE 12-5-3
DELAYED NEUTRON FRACTIONS OF VARIOUS FUELS

Fuel	β
^{233}U	0.0026
^{235}U	0.0065
^{239}Pu	0.0021

the reactor is decreasing in power level and neutron flux. The positive root, Equation 12-5-6, does not apply when Δk is less than 0.

Suppose a reactor is operating at a high power level. Suddenly the reactivity is decreased by a relatively large amount. There is a sudden drop in power level and a decrease in neutron flux. The delayed neutrons originating from those fissions that occurred before the decrease in reactivity continue to be created for quite some time. The delayed neutron emitters with extremely short half-lives soon disappear, and shortly the only neutrons remaining are those contributed by ^{87}Br with a 55-second half-life. Under these conditions, the period becomes approximately minus 80 seconds. Figure 12-5-4 shows the change in neutron flux as a function of the time following a negative step change in reactivity.

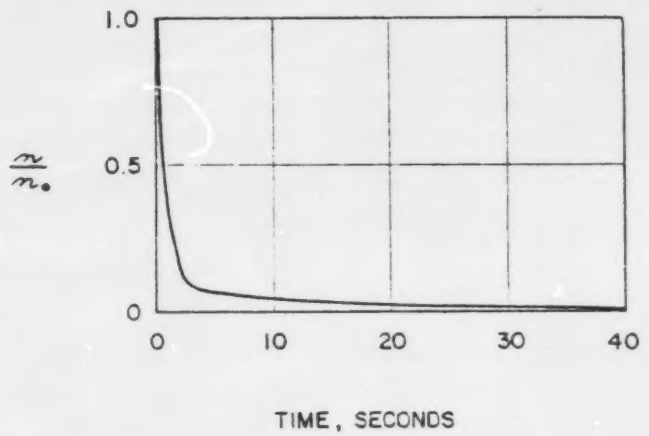


Figure 12-5-4. Reactor response to a negative step change of reactivity. (From Bennet, page 172.)

EXERCISES

1. A reactor is operating at a constant level with k_{eff} = 1.000. A piece of pure ^{235}U weighing 6 kilograms is inserted into the core. What would be the effect on k_{eff}?
2. What statement can be made about the value of k_{eff} for a subcritical training facility?
3. Why would a reactor be nearly impossible to control if there were no delayed neutrons?
4. Given the values for β in Table 12-5-2, which reactor would allow the operator the most time to make adjustments?
5. A reactor is scrammed by the rapid insertion of neutron absorbing rods, and found to have a period of minus 80 seconds. Can you show why it has this characteristic period?
6. A reactor is operating at 100 watts when an insertion of reactivity is made that produces a period of 25 seconds. What is the power level of the reactor 1 minute later? Two minutes later? Ten minutes later?
7. Using Figure 12-5-2, determine the reactivity for a reactor with a period of 20 seconds. Can this be done if the period is 0.1 second? If so, what is it, and if not, why not?
8. A new fuel, bolonium-287, has been discovered. After fissioning, it emits three groups of delayed neutrons with half-lives of 0.5, 1.5 and 63 seconds; β = 0.0064. What would you expect the period to be following a scram?
9. A reactor uses water as the moderator and has a period of 6 seconds under certain conditions. If the moderator is changed so that the average diffusion time is doubled, what would happen to the period? (Hint: See Equation 12-5-6.)

MATERIALS

Reactor.
Semi-log graph paper.
Stopwatch.

PROCEDURES

OBSERVATION OF THE EFFECT OF DELAYED NEUTRONS FOLLOWING A STEP INCREASE IN REACTOR POWER LEVEL

Raise the control rods to bring the reactor critical at a power level less than 5% of full power. By adjustment of the control rods, hold the reactor at this power level for at least 5 minutes and until both the control rod positions and power are constant.

Raise the control rods as quickly as prudently possible until the power level is about 20% of full power. Record in Data Table 12-5-1 either the stable period or the time necessary to make the approximate step increase in power.

Upon reaching the desired power, the operator should hold at that power level by moving only one rod (with a position indicator) back into the core.

When the upper power level is reached, the student should start the stopwatch and fill in part D of Data Table 12-5-1.

After there ceases to be any significant change in the control rod position, insert the control rods and lower the reactor power to the initial starting power level (A on Data

Table 12-5-1). Hold the power level there for at least 5 minutes and until the control rod positions are constant.

Repeat the above step insertion, only adjust the rate of control rod removal to achieve a period or length of time which is twice as long as the first increase. Fill the data in as before on Data Table 12-5-2.

Both sets of data will be treated in an identical manner. Call the fuel rod position at the end of each experiment P_o. Subtract P_o from each position listed in part D of the Data Tables. These rod positions represent the excess rod which was out of core at that time. Clearly, at the end of the experiment there is zero excess rod out of core. Plot the log of excess control rod position as a function of time: this can be easily accomplished on semi-log paper.

Using a straight edge through the data points with the largest time values, isolate the delayed neutron precursor with the longest half-life. Draw a straight line through these points. Determine the half-life associated with this line. It should be about 55 seconds.

Compare the two curves and then answer the following questions:

1. Why does the control rod have to continue to be driven in after the reactor is made critical at the upper power level?
2. Compare the length of time required for the control rod to reach the minimum positions after the step increase; explain the difference.
3. Why does the slope at the line which represents the end of the curve correspond to about 55 seconds?

THE OBSERVATION OF DELAYED NEUTRONS FOLLOWING REACTOR SHUTDOWN

Bring the reactor to criticality at any power greater than 50% of full power. After at least 5 minutes, when both the power and control rod positions are stable, drop (scram) or drive the rods into the reactor as quickly as possible. Follow either the power level or the neutron flux down until it ceases to diminish for at least 1 minute. This may take some time.

Fill out Data Table 12-5-3 either at the appropriate time or by quantitatively analyzing a strip chart recorder graph after the experiment.

Subtract the background count rate or power level from each count and place the net in the third column of the Data Table. Plot the log of the count rate against time on semi-log paper.

Draw a straight line through the points associated with the largest time values. Determine the half-life associated with this line. Graphically subtract the values of the straight line from each of the data points. Attempt to draw another straight line through these points. Determine the half-life associated with the second straight line.

Compare the two half-lives of the straight lines with the half-lives of the two longest-lived, delayed neutron precursors found in Table 12-5-1.

DATA TABLE 12-5-1

A. Stable power before step increase _____seconds.

B. Stable power following step increase _____seconds.

C. Period or length of time to make the step increase _____seconds.

D.

Time After Reaching Power Level B	Control Rod Position	Control Rod Position - P_o
0		
5		
10		
15		
20		
25		
30		
35		
40		
45		
50		
55		
60		
70		
80		

DATA TABLE 12-5-2

A. Stable power before step increase _____seconds.

B. Stable power following step increase _____seconds.

C. Period or length of time to make the step increase _____seconds.

D.

Time After Reaching Power Level B	Control Rod Position	Control Rod Position - P_o
0		
5		
10		
15		
20		
25		
30		
35		
40		
45		
50		
55		
60		
70		
80		

DATA TABLE 12-5-3

Time After Full Insertion of Control Rods	Power or Neutron Count Rate	Net Power or Neutron Count Rate = A - Background
10 seconds		
15		
20		
25		
30		
35		
40		
45		
50		
55		
60		
70		
80		
90		
100		
110		
120		

Bennet, D.J. The Elements of Nuclear Power. New York: John Wiley & Sons, 1973.

Glasstone, Samuel and Sesonske, Alexander. Nuclear Reactor Engineering. New York: D. Van Nostrand, 1967.

LaMarsh, John R. Introduction to Nuclear Engineering. Reading, MA: Addison-Wesley Publishing Co., 1975.

_____. Introduction to Nuclear Reactor Theory. Reading, MA: Addison-Wesley Publishing Co., 1966.

Murray, Raymond L. Nuclear Energy. New York: Pergamon Press, 1975.

Pedersen, Knud B., et al. Applied Nuclear Power Engineering for Practicing Engineers. Boston: Cahners Books, 1972.

1. Define reactor period:

2. Give a typical value for a prompt neutron generation lifetime:

3. Explain why a reactor is uncontrollable by ordinary methods when operated only on prompt neutrons:

4. Explain, by example, effect of delayed neutrons upon reactor behavior:

5. List typical values of k_{eff} and β for subcritical, critical and supercritical reactors:

6. Briefly explain concept of reactivity:

7. Given a reactivity-versus-period graph, find reactivity corresponding to any specified period:

8. Briefly define or explain:

   Δk (excess multiplication factor).

   Prompt neutron lifetime (ℓ_p).

Delayed neutrons.

β.

In-hour equation.

Reactor period.

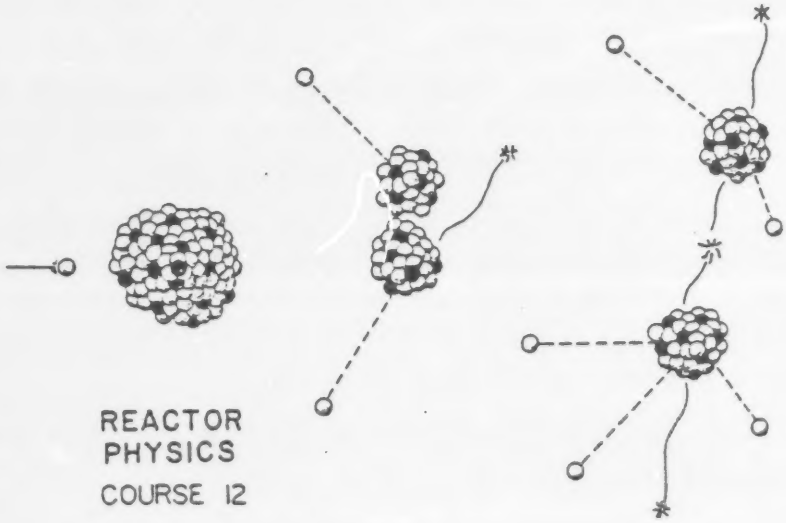
Prompt critical.

In-hour curve.

NUCLEAR TECHNOLOGY

MODULE 12-6 REACTIVITY - FUEL MODERATOR AND CONTROL SYSTEMS. -

RONALD L. KATHREN AND LARRY B. CHURCH



TERC-SW

TECHNICAL EDUCATION RESEARCH CENTER — SOUTHWEST
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COURSE 12 REACTOR PHYSICS

MODULE 12-1 Neutron Physics

MODULE 12-2 Cross Sections and Reactions

MODULE 12-3 Fission Rate and Reactor Power

MODULE 12-4 Time Independent Nuclear Theory

MODULE 12-5 Time Dependent Nuclear Theory

MODULE 12-6 Reactivity - Fuel Moderator and Control Systems

MODULE 12-7 Internal Effects on Reactivity

MODULE 12-8 Reactor Operating Characteristics

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INTRODUCTION

In any nuclear reactor, the fuel and control-rod systems are vital components. The relationship of fuel and control systems primarily determines the extent of nuclear chain reaction. Other systems have generally lesser, although equally important, control functions. In a thermal reactor - one requiring thermal neutrons to achieve a self-sustaining chain reaction - the moderator is also of considerable importance. Methods and philosophy for achieving a sound fuel management program, efficient moderation and safe control of the reactor are presented in this module, which also describes basic strategy for guiding the operation of any reactor system.

MODULE PREREQUISITES

The student should have completed Course 1, "Radiation Physics," and an elementary college freshman level course in chemistry or physics.

OBJECTIVES

Upon completion of this module, the student should be able to:

1. State what is meant by core excess maximum available reactivity and demonstrate why this is an important core parameter.
2. Describe how ^{239}Pu is produced in a typical uranium-fueled reactor.
3. Explain purpose of a moderator.
4. Give chemical descriptions of several common moderators.
5. Show why materials listed in Objective 4 are good moderators.
6. Describe moderator effect on reactivity.
7. Show why enrichment is necessary when using light water as the moderator.
8. Show why enrichment is not necessary when using heavy water as the moderator.
9. Diagram flux around a control rod.
10. State effect of fuel burnup on reactivity.
11. State means of compensating for burnup.
12. Define:
    - Core excess.
    - Heavy water.
    - Light water.
    - SCRAM.
    - Megawatt day (MWd).
    - Burnable poison.
    - Chemical shim.
13. Calibrate worth of a control rod (on a training reactor); state significance of results.

CORE EXCESS AND REACTIVITY

One of the most important and frequently measured parameters of a nuclear reactor is the core excess. When a reactor is critical at low power, the core excess is a measure of the resultant reactivity if all control rods and chemical poisons were removed. Core excess represents the amount of reactivity worth of control rods and/or chemical poison that must be kept in the reactor to keep it exactly critical (i.e., at $k_{eff} = 1.000$). The core excess may be measured at any power level, but the normal reference point is termed "cold free of Xe," which refers to the reactor at normal room temperature (hence the "cold") and "free of Xe" means that there is no (or a negligible amount) of ^{135}Xe, a fission product that is produced during reactor operation. Xenon-135 free is an important consideration because ^{135}Xe has a high thermal neutron cross section that acts as a poison. If present in the reactor it will affect the reactivity and give an erroneous picture of core excess. The effects of both temperature and Xe poisoning will be discussed in detail in Module 12-7, "Internal Effects on Reactivity."

An important point to realize about core excess is that it represents the maximum possible reactivity insertion. With the rods in place and the reactor critical, the core excess is a measure of the worth of the rods still remaining in the reactor; hence, if they are suddenly pulled out, an amount of reactivity equal to the core excess would result. The core excess is a strong function of the quantity of fissile fuel in the reactor. The more fuel, the

more control-rod insertion necessary to keep the reactor exactly critical and the larger the core excess.

FUEL

For the purpose of this discussion only, uranium-fueled reactors will be considered because almost all commercially available reactors in the U.S. are uranium-fueled. Although ^{239}Pu is available in quantity for use as a reactor fuel, the Nuclear Regulatory Commission does not at this time license the use of ^{239}Pu as a reactor fuel, nor does it anticipate the issuance of such licenses until about 1985.

For reactor fuel, the uranium may be in the form of a metal, in an alloy with other materials, or as dioxide or carbide. It is usually covered ("clad") with stainless steel or aluminum, magnesium or zirconium alloy, much like the "tin can" used to can food. This is to protect the fuel from attack by its environment (cooling material) and to prevent the escape of fission products (most of which are highly radioactive).

In a thermal reactor there must be present at least one of the three fissile species: ^{233}U, ^{235}U or ^{239}Pu. Naturally occurring uranium contains 99.28% nonfissile ^{238}U; the remainder, 0.72%, is the fissile ^{235}U. However, ^{239}Pu can be obtained from ^{238}U as shown by Equation 12-6-1:

$$^{238}_{92}U + ^{1}_{0}n \rightarrow ^{239}_{92}U \xrightarrow[t_{1/2} = 23\ min]{\beta^-} ^{239}_{93}Np \xrightarrow[t_{1/2} = 2.3\ days]{\beta^-} ^{239}_{94}Pu.$$

Equation 12-6-1
(Equation = Eq.)

On the other hand, ^{233}U is produced by neutron capture of ^{232}Th, which is used in high-temperature, gas-cooled reactors.

To increase the fuel utilization factor of the six-factor formula, the United States has always chosen to use a program that has favored ^{235}U enrichment of the raw uranium. Other nations, such as Canada, without ^{235}U isotope separation capability, have chosen to use natural uranium as the fuel. However, as will be described, moderators other than H_2O must then be used to achieve reactor criticality. The choice is one of using natural uranium (with a low fuel utilization factor) and to make up the difference by increasing the thermal utilization factor with special moderators, or to use enriched fuel with its high fuel utilization factor. This permits the use of common water (H_2O) which has a relatively low thermal utilization factor as the moderator. The difference is the overall cost budget. Using nonenriched fuel, the uranium is relatively cheap, but the special moderator is expensive. On the other hand, using enriched fuel will result in higher fuel costs but a concurrent savings from using the cheaper common water as moderator.

Partially enriched uranium, containing from 1 to 5 percent ^{235}U, is used as the fuel in most power reactors in the U.S. In contrast, higher enrichment material is usually used in special reactors such as research reactors and, in very, very high enrichments, for nuclear weapons. Highly enriched uranium is sometimes known as oralloy, a term derived from the Manhattan Project days when secrecy was required. The maximum enrichment is about 93% ^{235}U. Unenriched or natural uranium is sometimes called tuballoy (pronounced tube alloy) and uranium from which the ^{235}U

has been removed is known as depleted uranium or D-238.

The primary method used for enrichment of uranium in the ^{235}U isotope is gaseous diffusion of UF_6. This method makes use of the different rates at which the gaseous fluorides of uranium isotopes diffuse through a porous barrier. The barriers used in the uranium isotope separation contain millions of pores per square centimeter. The lighter molecular weight materials, associated with the lighter ^{235}U, will diffuse through the pores at a faster rate than the heavier material associated with the ^{238}U. Thus, the gas that passes through the barrier will be slightly more enriched in ^{235}U compared to ^{238}U.

The extent of separation per barrier is called the separation factor which is equal to the square root of the ratios of the molecular weights. As is seen in Equation 12-6-2 the separation factor for ^{238}U compared to ^{235}U is very close to 1.00000, which signifies no separation.

$$\text{Separation factor} = \sqrt{\frac{\text{molecular weight of } ^{238}UF_6}{\text{molecular weight of } ^{235}UF_6}}$$

$$= \sqrt{\frac{352}{349}} = 1.0085.$$

Eq. 12-6-2

As a result, a long string of barriers must be constructed; often such a cascade of barriers consists of thousands of stages. The larger the number of stages, the higher the

final degree of enrichment. In the design of a diffusion plant, the increased enrichment and percentage recovery resulting from an increase in the number of stages must be weighed against the cost of building and operating the plant.

When placed into the fuel element, the uranium-containing material should have the property of allowing a high percentage of the heat generated to be released to the cooling material. This requires a material with a high thermal conductivity. The material should also be highly resistant to radiation damage lest the life of the element be dictated by the physical and chemical disintegration of the fuel, rather than by actual fuel burnup or utilization.

The fuel of choice is usually uranium dioxide, UO_2. As seen in Table 12-6-1, UO_2 appears to have the best com-

TABLE 12-6-1
PROPERTIES OF URANIUM-CONTAINING COMPOUNDS

PROPERTY	U METAL	UO_2	URANIUM CARBIDE
Susceptibility to radiation damage	Poor	Very good	Very good
Thermal conductivity	Very good	Poor	Very good
Corrosion resistance to air or water	Poor	Very good	Poor
Temperature stability	Very poor	Very good	Very good

bination of physical properties of all the uranium compounds that can be economically made from UF_6. The major problem with UO_2 is its low thermal conductivity, although this is partially offset by the fact that very high temperatures are permissible in the center of the fuel element.

The material into which the uranium compound is placed is called the fuel element cladding. Cladding must be able to withstand corrosion attack by both fuel and coolant at the high core temperatures and to withstand the radiation field existing within the core without undergoing degradation. For most water-cooled reactors the preferred cladding materials are aluminum, stainless steel and Zircaloy. The last is an alloy of zirconium and steel; it is more expensive, but this difference in price is offset by the very low thermal neutron capture cross section of zirconium.

The diameter of a fuel rod is dependent upon several variables. A small diameter fuel rod will have superior heat transfer characteristics compared to a larger diameter one. On the other hand, the smaller diameter rod will be more expensive to manufacture and has a greater likelihood of undergoing cladding failure because of increased surface area of cladding in contact with the coolant for a given fuel mass. On the other hand, a larger diameter rod will decrease the neutron escape probability and increase the probability of conversion of ^{238}U to ^{239}Pu.

One common power reactor using slightly enriched fuel (2 to 4%) has fuel rods with an outside diameter of 0.3 inch and a stainless steel cladding of 0.02 inch. These rather narrow rods are 90 inches (7.5 feet) long and are collected together in groups of about 300, each such group being a fuel bundle. Several bundles may comprise a fuel assembly or element. Table 12-6-2 shows percentage of various materials in the core of such a reactor.

TABLE 12-6-2
PRINCIPAL MATERIALS IN A TYPICAL REACTOR CORE

Material	% by Volume
Uranium dioxide	35
Water	50
Stainless steel	12
Zircaloy	3

In considering fuels it is important to appreciate the role of ^{239}Pu; there are two ways in which its fissile properties may be used. In a breeder reactor, depleted uranium (uranium that has had all of the ^{235}U removed) is placed near the core in what is called a blanket. Periodically, the blanket is removed and the ^{239}Pu generated is chemically separated and made into fuel. A breeder reactor not only generates power of its own, but also generates fuel for the future. At the present time the economic viability, safety and environmental concerns of a breeder system are not clear and the first large scale breeder facility in the U.S. is not scheduled for completion before the mid 1980's, if at all. However, breeder reactors are being operated in other countries at the present time.

Plutonium-239 is being used within power facilities without chemical separation. These converter reactors start with a 2 to 4% enrichment of ^{235}U and initially are completely dependent upon this fissile nuclide for power. As the ^{235}U is consumed within a fuel rod, ^{238}U is converted to the fissile ^{239}Pu. As reactor use continues, a greater

fraction of the reactor power is generated by the fission of ^{239}Pu, and conversely a smaller and smaller fraction of the power comes from the fission of ^{235}U. Eventually, however, the fuel must be replaced, for more ^{235}U is consumed than ^{239}Pu produced for replacement. All reactors containing ^{238}U produce some plutonium.

MODERATOR

The purpose of the moderator is to slow down the high-energy neutrons created at the time of fission to energies low enough to efficiently cause other fissions. Neutrons are created by the fission process with a rather wide spectrum of energies, the average being close to 2 MeV (Figure 12-6-1). The job of the moderator is to reduce these neutron energies to the thermal range of 0.025 eV; the desirability

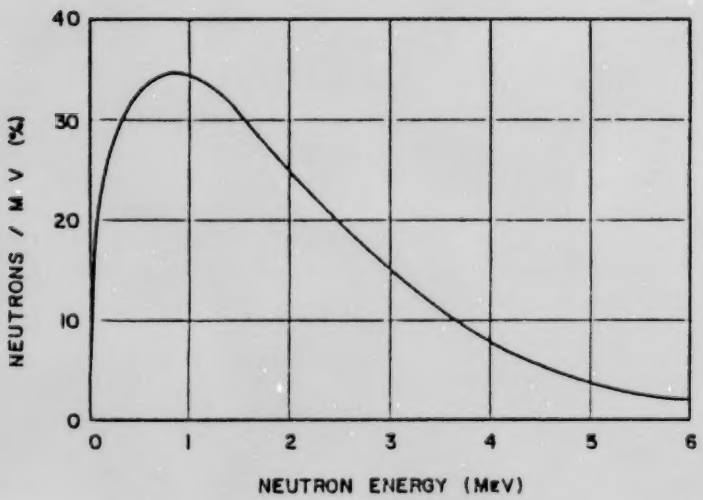

Figure 12-6-1. Fission neutron spectrum.

of this (Figure 12-6-2) is to take advantage of the great-

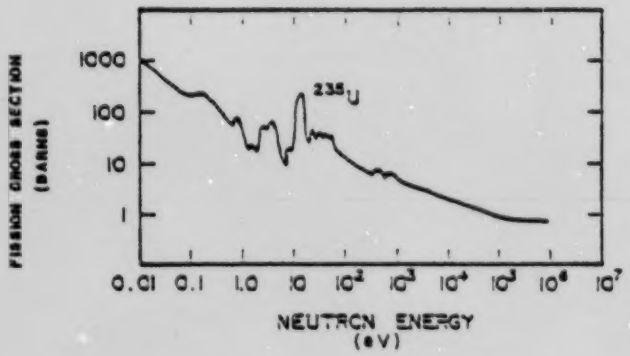

Figure 12-6-2. Fission neutron cross section for ^{235}U as a function of neutron energy.

ly increased fission cross section at lower energies.

Efficient moderation is accomplished through elastic collisions between neutrons and similar sized moderator atoms. When an energetic particle strikes another particle, maximum energy is removed from the energetic one when the two masses are equal. Thus, hydrogen atoms (atomic weight of about 1.000 amu) will most efficiently remove the energy (or moderate) neutrons (also of atomic weight of about 1.000 amu). The maximum fractional energy lost in a single elastic collision is inversely proportional to the difference in mass between the two collision partners.

Another important attribute for good moderation is the necessity of a low capture cross section. To be used in the moderation of fission neutrons, the moderator cannot capture (that is effectively remove) many neutrons before they reach optimum thermal energies.

In summary, the three most important properties of a

good moderator are 1) a high elastic scattering cross section, 2) the average energy loss per collision, and 3) a low neutron capture cross section. The first is usually found only in low mass elements; the last cannot be easily predicted as a function of atomic mass. It is also desirable for the moderator to have a large thermal conductivity so it does not heat up in the core. It must be able to withstand intense radiation without physically disintegrating. Table 12-6-3 gives neutron scattering and absorption cross sections

TABLE 12-6-3
PROPERTIES OF COMMON MODERATORS

Compound or Element	Macroscopic Cross Section, cm^{-1}		Moderating Ratio
	Scattering, Σ_s	Absorption, Σ_a	
H_2O	1.03	0.66	58
D_2O	13.6	0.003	21,000
He	0.8	0.007	45
Be	7.0	0.01	130
C	4.8	0.0034	200

for the major low mass elements. The ratio of the slowing down power to the macroscopic absorption cross section is the moderating ratio. This ratio is a measure of the usefulness of the material as a moderator. The larger the moderating ratio, the better the material for neutron moderation. The moderating ratio is calculated from macroscopic

cross sections for scattering and absorption.

Despite its rather unimpressive moderating ratio, H_2O is perhaps the most popular moderator; this is in part due to the low cost of purification and availability. Because of its relatively high absorption cross section, a critical system cannot be obtained using light water as the moderator unless the fuel has undergone ^{235}U enrichment. Also, the decomposition of water by high-intensity radiation with the liberation of hydrogen and oxygen can sometimes create a problem.

Because of its excellent moderating ratio, heavy water (D_2O) is an excellent moderator. (Light water, also used in nuclear reactors, is water in which both the hydrogen atoms in each molecule are of the isotope protium.) In fact, a system of natural uranium and heavy water can become critical; this system is used in the Canadian CANDU power reactors. While heavy water is available, its cost is quite high.

A rather unusual moderator is zirconium hydride (ZrH_2). Because the zirconium capture cross section is very small, the moderating ratio of this material approximates pure hydrogen. In the fuel element, ZrH_2 is alloyed with the uranium fuel. Hydrogen atoms are bonded to the zirconium atom and their moderation ability is inversely proportional to temperature. The higher the temperature, the poorer the ability of the hydrogen to moderate neutrons. Because of the close contact between the ZrH_2 and the fuel, as the fuel heats up, the ability of the hydrogen to moderate decreases. This property results in an unusual degree of reactor stability, and suggests its use in training and research reactors. (The TRIGA type reactors use this principle.) Because the uranium-zirconium hydride alloy has

a low thermal conductivity, such moderator systems cannot usually be used in power facilities where higher temperatures are encountered.

CONTROL RODS

Generally speaking, a reactor has three different types of control rods, each serving a different function. Course control of the reactor system is achieved with the use of shim rods. They are used to bring the reactor approximately to the desired power level when the system is started up. The shim rods must, therefore, have a fairly large reactivity equivalent or worth. When the desired power level of the reactor is close, control is taken over by the regulating rods. The rods must be able to maintain the reactor at a constant power and counteract the effects of rapid or transient changes. The worth of the rods is small, but they should be capable of moving rapidly to provide the necessary quick response. Lastly there are safety rods, which shut down the reactor quickly in the event of an emergency. Such a movement is called a SCRAM. Safety rods, therefore, must be able to move quickly as well as have a large worth; the net worth of the safety rods must exceed the maximum core excess of the reactor. These rods are comparable to an emergency brake in a car; they do not control the speed of the car or the power of the reactor. It is either applied all the way or shut off all the way, either full up or full down, and they can be used to make an emergency stop of the car or an emergency SCRAM of the reactor.

The most fail-safe way to have a control rod return to the core is through the use of gravity. When a SCRAM occurs, the device holding the rod out of the core is de-energized and the rod drops quickly into the core. Often the safety rods have fuel attached to their bottoms; these are called fuel-followers. In a SCRAM situation, not only are the control rods being placed in the core, but some fuel drops out of the core configuration to further reduce reactivity.

The only purpose of control rods is to aid in safe control of reactor flux. Removal or introduction of a control rod influences the thermal utilization factor of the six-factor formula. The primary criterion for the design of a reactor control rod is its ability to absorb neutrons, hence elements are selected with large neutron-capture cross sections. The most popular elements are boron, cadmium, hafnium, indium and silver. The reaction of interest with boron is the $^{10}B(n,\alpha)^{7}Li$ reaction; the rest are the more traditional (n,α) capture reactions.

Materials used in control rods must be able to withstand very high radiation environments and temperatures. Under these conditions materials should not experience any loss of strength or undergo physical or dimensional changes. Since heat is often generated in the control rods, a high thermal conductivity is also desirable. Boron, either natural or enriched in ^{10}B, is generally used as an alloy or as a powder clad in stainless steel. A problem associated with use of boron for reactor control is the liberation of helium gas by an (n,α) reaction, which can cause swelling and cracking of the material or assembly.

A power facility is usually designed to run with the control rods completely removed from the core. This is be-

cause control rods remove neutrons from the neutron pool and a power facility is designed to produce neutrons for the least possible cost. Removal of any neutrons decreases the efficient manner of operation.

Another problem associated with running a reactor with only partial insertion of control rods is the nonuniform flux distribution near the control rod (Figure 12-6-3).

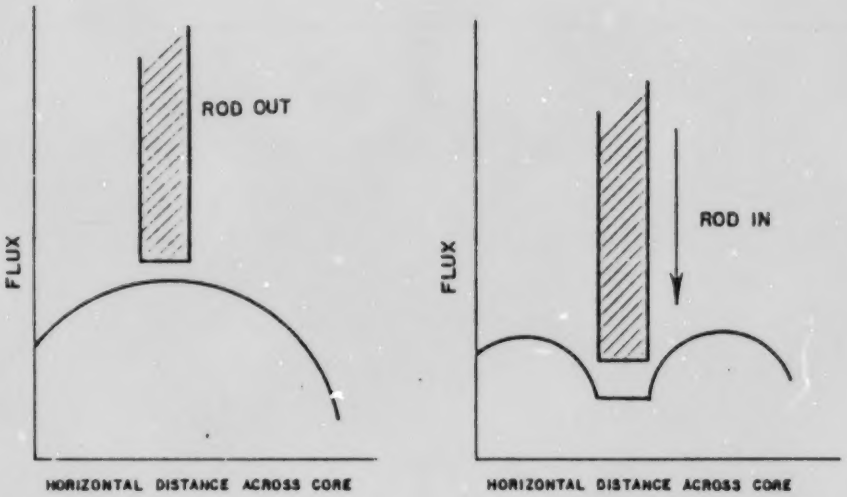

Figure 12-6-3. Flux distributions with a central control rod in and out.

The control rod diminishes the extent of the neutron flux in the region near it, amounting to what is known as non-even fuel burnup; fuel elements near the control rod do not have their uranium removed or used up as quickly as elements further away from the control rod.

The unit of fuel exposure is the megawatt day (MWd), equal to the reactor power level in megawatts times number of days at that power. Thus elements near the control rod

or in its "shadow" may be recorded as having a fuel exposure of X MWd; but if the average flux at that position were only half what the average of the rest of the core was, then the true exposure on the fuel element would only be 0.5X MWd. At the end of a fuel cycle, this element (along with the rest of the elements in a given core region) would be removed with one-half of its useful life still remaining. Such a practice is not very good economic practice and would be indicative of a poor fuel management program.

The desire of a good fuel management program is to have all the fuel elements in the core or region of the core burn as evenly as possible, both as a function of geometry and also as a function of time. The latter is desired so as to force the minimum number of fuel changes. One way of "flattening out" the time versus fuel burnup curve is with the use of a burnable poison. For example, a compound of boron may be included within the fuel element as a poison. The poison captures neutrons and is thereby consumed or burned up during operation of the reactor. Consequently, the amount of poison decreases with time and its negative reactivity is diminished as the reactor fuel is decreased. Ideally, the burnable poison is consumed at a rate that exactly matches the consumption of fuel; the resultant core excess does not decrease as a function of time.

A common way to control a reactor without use of control rods is with an element such as boron added to the coolant water - often called a chemical shim. To decrease power level, the coolant water boron concentration is raised. An increase in power involves chemically removing boron from the coolant water. This approach to control has the advantage over control rods of ensuring a more uniform geometrical flux distribution throughout the core; there can be

no dead-spots (relatively unused fuel) at the end of a fuel cycle.

This approach to increasing power must be done slowly, however, because of the relatively long time required to remove boron from the coolant, when compared to the time needed to raise a control rod. On the other hand, very often large volumes of highly borated water are available to be released into the core in the case of an emergency to quickly lower the reactivity.

EXERCISES

1. The core excess of a cold, Xe-free research reactor is measured as $3.41. An experimenter wants to run a small sample of borated aluminum into the core for an irradiation study. With the sample in the core, the core excess is $3.16. What is the reactivity of the borated aluminum?
2. If the core excess of a cold reactor is $3.50 and of a hot (600°F) reactor $3.02, what effect does temperature have on reactivity?
3. Why must D_2O rather than H_2O be used in a natural uranium-fueled reactor?
4. What are the advantages of enriched fuel? What one disadvantage offsets these?

MATERIALS

Access to a training reactor which has either of the following:

An accurate period meter in the 30-second range.

Linear power meter to be used with stopwatches to calculate period.

Linear graph paper (Cartesian coordinates).

PROCEDURES

BACKGROUND

The purpose of this procedure is to participate in calibration of a control rod, which should be done at a research or training reactor. Not everyone will be able to individually calibrate a control rod; therefore, calibration should be done as a group exercise. Everyone is encouraged to take all the raw data and to make necessary calculations for the desired graphs, and to summarize linear loss results.

Before commencing the exercise it might be necessary to restudy those parts of Module 12-5, "Reactivity - Fuel Moderator and Control Systems," that pertain to reactor period, reactivity and the relationship between period and reactivity (the in-hour equation).

It is best to start with the calibration of a regulating rod. Often, the full calibration of a shim or safety

rod cannot be accomplished because the reactor will not go critical when it is full into the core. If the reactor has an accurate period meter in the range of 30 seconds, then that may be used; if it does not, then a linear power channel should be used in conjunction with stopwatches. Time necessary to double the reactor power will be taken and this time may be converted into the reactor period through use of the following equation:

$$T = t_d/0.69315 \qquad \text{Eq. 12-6-3}$$

where

T = reactor period.

t_d = doubling time.

0.69315 = constant which is the natural log of 2.

DATA COLLECTION

1. Establish a critical condition at a very low power chosen by instructor (depending upon the reactor, this whould be in the vicinity of 5 watts). Leave the regulating rod to be calibrated inserted fully into the core. Allow the power level to stabilize for at least seven minutes; this will permit the delayed neutrons to come to full equilibrium. Record all rod positions on Data Table 12-6-1.
2. (Repeated doubling times should be measured during

source excursion instead of just once. In this manner, the asymptotic period can be identified. Excursion should probably be started at lower power level to extend range before expansion feedback limits range of asymptotic period. Period must be measured between levels at which source effects distort results and levels at which temperature reactivity feedback occurs.) Withdraw regulating rod to be calibrated so as to get a transient period of about 15 seconds; this withdrawal should be done all at once and then stopped. The stable period should be in the range of 25 to 30 seconds. Allow the power level to increase for about a minute to let the transient period decay out completely. If an accurate period meter is available, record the period. If one is not available, use a stopwatch to record the doubling time (30% to 60%, or 40% to 80%) of the linear power range. If stopwatches are being used, take several readings over several different ranges and then average the doubling time. Record the data.

3. Return reactor to the previous low power by inserting a shim, safety or other regulating rod. Do not reposition the regulating rod undergoing calibration. Also, do not let power of the reactor build up to a point where temperature effects come into play (discussed in more detail in Module 12-7, "Internal Effects on Reactivity"). After seven minutes or so to allow the power to stabilize, repeat above procedure with another pull of the regulating rod. Be sure to record all the necessary raw data: rod positions and periods or doubling times. Continue this procedure until the regulating rod is completely withdrawn from the reactor.

4. Calculate the following data from the raw data: period, if necessary, using the average doubling time and equation given above, the in-hour equation (Equation 12-6-5 below), or a computer printout of the in-hour equation, calculate the ratio of reactivity to beta, $\Delta k/\beta = \$$, for the withdrawn section; in the next column of the data sheets, sum the worth of the previous column so as to show the integral worth of the rod to that point of withdrawal. Obtain a graph of reactor period versus reactivity $\Delta k/\beta$ in dollars for the facility upon which this procedure is run. A typical graph ($\beta = 0.0073$) is shown in Figure 12-6-4.

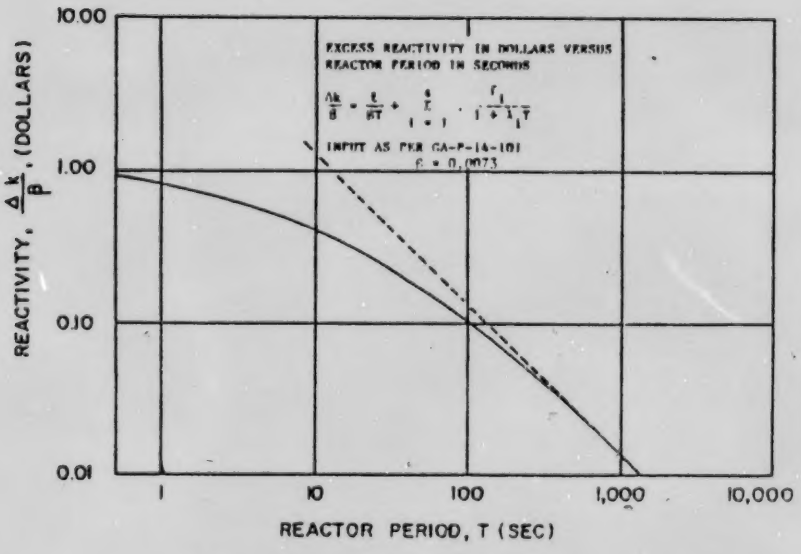

Figure 12-6-4. A typical reactivity versus reactor period curve, $\beta = 0.0073$.

The in-hour equation:

$$\$ = \frac{\Delta k}{\beta} = \frac{\ell}{\beta T} + \sum_{i=1}^{6} \frac{f_i}{1 + \lambda_i T}$$

Eq. 12-6-4

where:

ℓ = prompt neutron lifetime.

T = period.

f_i = fraction of i'th group of delayed neutrons.

λ_i = decay constant of the i'th group.

5. To see the differential form of the rod worth as a function of position, plot (on linear graph paper) the regulating rod position against the reactivity for each withdrawn section. Explain why the curve has the shape that it has.

6. Plot rod position against total worth at that position to obtain the integral rod worth curve. To obtain a graph that will be usable, the reactivity axis will have to be reversed: when rod is full out, there should be zero rod worth left in the core, and when the rod is fully inserted, the full rod worth will be inserted in the core. By using this newly constructed curve, rod position will indicate the remaining rod worth left in the core; in a critical condition the sum of these rod worths is the core excess.

DATA TABLE 12-6-1

ROD CALIBRATION

Type reactor _____

Rod to be calibrated _____

Date _____

Low power level = _____

Rod Withdrawal No.	Rod Position After Withdrawal cm	Doubling Time, Sec. t_d	Period $T_{sec^{-1}}$	Reactivity $\$ = \Delta k/\beta$	Sum of $ (Integral Worth)	Other Rod Positions at Low Power After Withdrawal
0	Down	00	00	$0.00	$0.00	
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						

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Glasstone, Samuel. Sourcebook on Atomic Energy. 3rd ed. New York: Van Nostrand-Reinhold, 1967.

Glasstone, Samuel and Sesonske, Alexander. Nuclear Reactor Engineering. Princeton, NJ: D. Van Nostrand, 1967.

Grant, D.J. Elementary Reactor Physics. Oxford: Pergamon Press, 1966.

LaMarsh, John R. Introduction to Nuclear Engineering. Reading, MA: Addison-Wesley Publishing Co., 1975.

Murray, R.L. Nuclear Energy. New York: Pergamon Press, 1975.

1. Briefly define core excess; demonstrate why it is a significant core parameter:

2. Briefly describe chemical and physical composition of a typical light-water fuel element:

3. Briefly describe how ^{239}Pu is produced in a typical uranium-fueled reactor:

4. Briefly explain purpose of a moderator:

5. List two common moderators:

   (1)

   (2)

6. Explain advantages and disadvantages of each of these two moderators:

7. Briefly explain why enrichment is necessary when light water is used as moderator:

8. Briefly explain why enrichment is not necessary when using heavy water as moderator:

9. Briefly describe moderator effect on reactivity:

10. Briefly explain what is meant by "chemical shim;" give one example:

11. Diagram the flux around a control rod, <u>in</u> and <u>out</u> of the core:

12. Explain effect of fuel burnup on reactivity:

13. Explain how to compensate for burnup:

14. Define:

Core excess.

Heavy water.

Light water.

SCRAM.

Megawatt day (MWd).

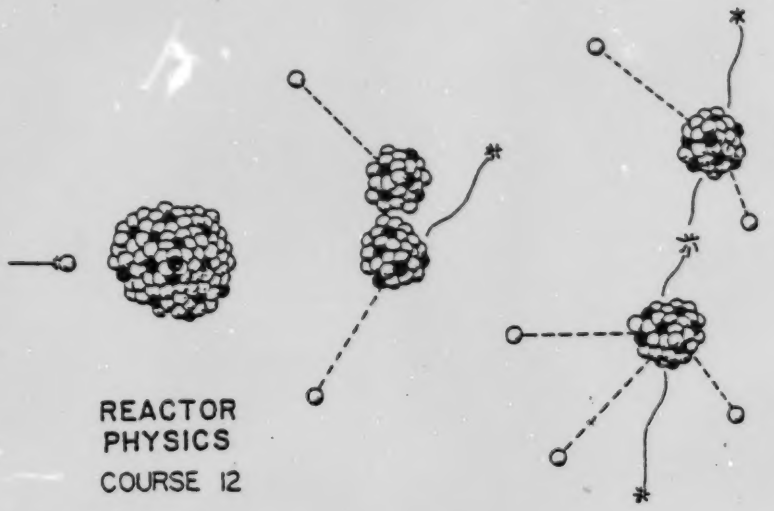
Burnable poison.

Chemical shim.

NUCLEAR TECHNOLOGY

MODULE 12-7 INTERNAL EFFECTS ON REACTIVITY, -

RONALD L. KATHREN AND LARRY B. CHURCH



REACTOR
PHYSICS
COURSE 12

TERC-SW

TECHNICAL EDUCATION RESEARCH CENTER — SOUTHWEST
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COURSE MODULES

COURSE 12 REACTOR PHYSICS

MODULE 12-1 Neutron Physics

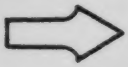
MODULE 12-2 Cross Sections and Reactions

MODULE 12-3 Fission Rate and Reactor Power

MODULE 12-4 Time Independent Reactor Theory

MODULE 12-5 Time Dependent Reactor Theory

MODULE 12-6 Reactivity - Fuel and Moderator



MODULE 12-7 Internal Effects on Reactivity

MODULE 12-8 Reactor Operating Characteristics

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INTRODUCTION

Many factors can affect reactor reactivity besides fuel, moderator or control system. Certain conditions within the reactor play roles in determining reactivity. Some are fixed, and relate to design or physical layout of reactor components. Others are variable, having to do with reactor operation.

In this module, three specific reactor-related factors are studied, all significantly affecting final reactivity:

Temperature.
Fission products.
Voids.

MODULE PREREQUISITES

The student should have completed the previous module in this Course, as well as its prerequisites.

OBJECTIVES

Upon completion of this module, the student should be able to:

1. Explain general effect of fission products on reactivity.
2. Write nuclear reaction equations for production and removal of ^{135}Xe.
3. State how ^{135}Xe effects reactivity.

4. Given an equation for rate of change of ^{135}Xe, define and give significance of all terms.
5. Plot ^{135}Xe concentration versus time for the following conditions:

   Xe free.

   Startup, immediately following full power at equilibrium Xe conditions.

   Startup at peak Xe condition.
6. Explain why some power reactors are not able to start up a few hours after shutdown.
7. List effect of ^{149}Sm on reactivity; give appropriate comparisons with ^{135}Xe.
8. Describe, in terms used in this module, the general effects of temperature on reactivity.
9. Explain how voids affect reactivity; give at least three typical void coefficients for existing nuclear reactors.
10. Show mathematically, as presented in this module, how temperature and void coefficients can be used to determine reactivity changes.

REACTIVITY COEFFICIENTS

Transient changes in the effective multiplication constant, k_{eff}, can be caused by changes within the internal conditions of the core. The core temperature, presence of voids or blank spaces in the moderator and inventory of fission products with high neutron cross sections are the three most significant internal conditions affecting k_{eff}. Changes in these parameters can affect the core as a whole or can be localized in a specific region of the core. How these changes affect k_{eff} - whether k_{eff} increases or decreases and how fast this occurs - is of great practical importance in reactor operations.

The degree or rate at which transient temperatures and void changes influence reactivity are expressed as coefficients of reactivity. Thus, the effects of temperature are expressed by the temperature coefficient and the effects of voids by the void coefficient. The unit for the temperature coefficient is the change in reactivity, $\Delta\rho$, per degree of temperature, or $\Delta\rho/°C$. Sometimes, the temperature coefficient is given as $\Delta\rho/\Delta T$, which is simply another way of characterization. The void coefficient is similarly expressed, having units of reactivity change, $\Delta\rho$, per fraction of the core volume replaced with air or steam. Void coefficients are frequently expressed as $\Delta\rho/\%$ void.

Reactivity coefficients can be either positive or negative, depending on their effect on the reactor. Thus, a reactor in which reactivity decreased as the temperature increased would have a negative temperature coefficient. All reactors licensed in the United States are designed to have negative temperature coefficients. Thus, as the core heats up

(or as the power increases) the reactivity goes down and the reactor tends to shut itself off. This is one of the inherent or built-in safety mechanisms in a nuclear reactor.

The same general consideration applies to voids. When some of the water in the core is replaced by air or steam, a void is produced. Thus there is a change in moderation which could affect reactivity, generally lowering it since there is less moderation. However, the water also acts as coolant, and without the coolant there is less material to remove the heat generated in the fuel. Hence, voids increase the risk of fuel overheating and melting or otherwise breaching the fuel cladding.

Although the temperature coefficient is correctly given in units of $\Delta\rho/\Delta T$, other units are sometimes used. This is because ρ may not be the driving unit of reactivity. Furthermore, fuel temperature is not a convenient measure of the loss of power. Instead, the loss in reactivity can be measured in dollars, \$:

$$\$ = \frac{\rho}{\beta} \qquad \text{Equation 12-7-1 (Equation = Eq.)}$$

where ρ is the reactivity (and equal to $\frac{k_{eff} - 1}{k_{eff}}$) and β is the delayed neutron fraction. The result is a prompt negative temperature coefficient expressed as loss of total reactor reactivity, -\$, per unit of power, usually expressed in megawatts, MW, or -\$/MW.

Temperature and void coefficients usually operate on a time scale that is very much slower than normal neutron fluctuations in the reactor. Reactivity effects from fission products are even slower. The relative time scales for normal neutron fluctuations, temperature and void effects and fission product

effects are seconds, minutes and hours, respectively.

TEMPERATURE EFFECTS ON REACTIVITY

As the power of a reactor changes, the temperature of the reactor changes; thus a feedback mechanism is established which has an important bearing on reactor safety. For example, an increase in power is followed by an increase in the temperature in the reactor. This causes decrease in reactivity, leading to a stable power level.

When a reactor undergoes a power increase, the temperature changes in different parts of the reactor at different rates. The fuel temperature will rise at nearly the same rate as the power increase, but the coolant temperature will rise much more slowly because of the time needed to transfer the heat from the fuel to the coolant. The temperature coefficients of the fuel, coolant and moderator must be considered separately; thus, the overall coefficient, α_T, is the sum of the three separate coefficients and can be shown mathematically:

$$\alpha_T = \frac{\Delta\rho}{\Delta T} = \frac{\Delta\rho}{\Delta T_f} + \frac{\Delta\rho}{\Delta T_c} + \frac{\Delta\rho}{\Delta T_m}. \qquad \text{Eq. 12-7-2}$$

In Equation 12-7-2, the total or overall temperature coefficient, α_T, is shown as the sum of change in reactivity per degree of the fuel ($\frac{\Delta\rho}{\Delta T_f}$), coolant ($\frac{\Delta\rho}{\Delta T_c}$) and moderator ($\frac{\Delta\rho}{\Delta T_m}$). Thus, α_T has units of reactivity per degree and, of course, can theoretically be either positive or negative. Note that,

in a light water reactor, the water acts as both coolant and moderator. Hence a single term, $\frac{\Delta\rho}{\Delta T_{H_2O}}$, may be used to replace to replace the last two terms in Equation 12-7-2.

An increase in the fuel temperature causes a broadening of the resonances in the absorption cross section of isotopes such as ^{235}U or ^{238}U. It increases the resonance absorption probability and thus decreases the resonance absorption escape probability; this decreases k_{eff}. The Doppler broadening causes the resonance escape probability to decrease with increasing fuel temperature, leading to a prompt negative temperature coefficient. Figure 12-7-1 shows the $^{238}U(n,\gamma)^{239}U$ cross section as a function of energy for three temperatures. Although the areas under the three curves are essentially constant, and hence the total cross section is constant, the fraction of neutrons which are able to escape through this energy region (the resonance escape probability) goes down as the temperature increases. The Doppler effect is caused by increase in random motion of the ^{238}U nuclei as they are heated.

The Doppler broadening produces a decrease in reactivity as temperature increases and, in some small reactors, can provide a negative temperature coefficient sufficiently great to protect against core overheating as well as providing for an effective upper limit to power excursions.

In a power reactor, the Doppler broadening also has a significant effect on cold startup. Because of the delay in transfer of heat from the fuel to the moderator, additional reactivity is needed to allow a cold reactor to operate at power, since the temperature coefficient of the fuel is negative. In a PWR, this is about 0.03%, and this additional increment of reactivity is attributed to the power Doppler effect, a term in common usage in the power reactor field.

In a thermal reactor, an increase in moderator coolant

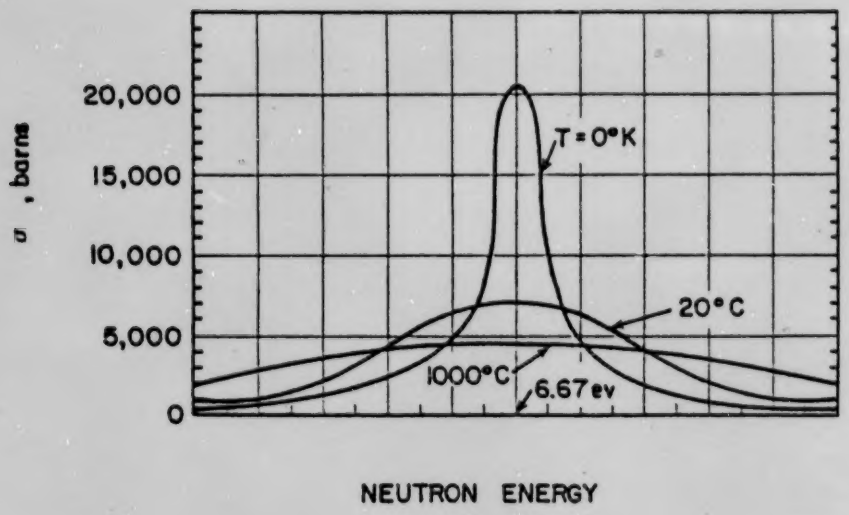

Figure 12-7-1. Doppler broadening of the 6.67 eV resonance capture peak of ^{238}U. (Adapted from Bennet, p. 226.)

temperature affects k_{eff} in two ways: 1) the physical density of the moderator changes due to thermal expansion, 2) the neutron energy spectrum is shifted to higher energies. The change in density is the more important of these two. Thermal neutron absorption by the moderator and coolant is lowered when their densities are reduced. Thus, the fraction of neutrons absorbed by the fuel relative to the moderator is increased. This means that the thermal utilization factor, f, increases with increasing temperature. This effect is especially pronounced when a chemical shim such as boric acid is present in the coolant-moderator, as some boron is expelled and made less dense along with the coolant moderator.

As neutron energies increase, the nonleakage probability decreases, and thus k_{eff} is decreased. In considering both of these effects on reactivity, due to the temperature co-

efficient of the moderator or coolant, it is important to realize that both of these occur with a speed which is very much slower than the much faster fuel temperature coefficient. Table 12-7-1 gives some typical temperature coefficients for some typical reactors. Generally, typical values range from

TABLE 12-7-1
REACTIVITY TEMPERATURE COEFFICIENTS AT 20°C

REACTOR	TYPE	OVERALL TEMP. COEFFICIENT (ρ/°C)
Brookhaven	Graphite Moderated	-4×10^{-5}
Calder Hall	British Graphite Moderated, Gas Cooled	-6×10^{-5}
Fermi	Fast Breeder	-1.8×10^{-5}
Shippingport	Pressurized Water	-5.5×10^{-4}
Dresden	Boiling Water	-6.7×10^{-5}

1×10^{-5} to 2×10^{-4} reactivity units per °C. Assuming an average of about -1×10^{-4} ρ/°C, a power reactor going from cold condition to power operation would show a 300°C temperature rise, and hence excess reactivity of -1×10^{-4} ρ/°C $\times$ 300°C = 0.03 would have to be supplied to overcome the negative temperature coefficient.

Note that the temperature coefficient of reactivity is in itself temperature dependent and tends to decrease (become more negative) with increasing temperature. In some reactors, the temperature coefficient may even be positive at low

temperatures and gradually decrease as the reactor heats up.

CHANGES IN REACTIVITY DUE TO VOIDS

When a reactor coolant liquid boils, the volume occupied by the vapor is essentially a void since the density of the vapor is very much less than that of the liquid. The void reactivity coefficient, α_v, is usually expressed as the change in reactivity per fraction void of the reactor core, X. In view of this definition, it follows that $\alpha_v = \Delta\rho/\Delta X$. If α_v is positive, then an increase in the void will lead to an increase in reactivity. This is obviously not a desirable situation because the temperature would increase, producing more boiling and more void. This could quickly lead to a situation where the reactor is uncontrollable. Hence, it is highly desirable for a reactor to have a negative void coefficient so that, if boiling does occur and a void results, the power will drop and the core becomes cooler, which is normally the case.

In fact, a negative void coefficient is easily achieved, since, when voids form in the moderator, the density of the moderator decreases sharply. With fewer atoms per unit volume, the ability of the moderator to slow down neutrons is decreased. Hence, the nonleakage probability is decreased because fewer neutrons are moderated, and k_{eff} (and hence reactivity) decreases. Note that the opposite will take place when the void collapses; hence, a reactor could undergo reactivity swings from the creation and collapse of voids. In general, however, the net effect is negative since the forma-

tion of the void first drives down the power level, and the reactor is cooler when the void collapses. Obviously, rapid void collapse is to be avoided.

Depending on the reactor design, the void coefficient can be either positive or negative, or can be positive at low temperatures and negative at high ones. In general, the void coefficient becomes smaller - that is, gets more negative - as the temperature of the reactor increases. Typical void coefficients are shown in Table 12-7-2. Obviously, the void

TABLE 12-7-2
TYPICAL VOID COEFFICIENTS AT OPERATING TEMPERATURE

REACTOR	TYPE	VOID COEFFICIENT, $\Delta\rho$/% VOID
Big Rock Point	Boiling Water	-1.7×10^{-3}
Humboldt Bay	Boiling Water	-2.3×10^{-3}
Fermi	Na Cooled Fast Breeder	No Voids Expected, but $\Delta\rho$/% Void Would be Negative
Candu	Heavy Water Moderated, Natural U Fueled	$+3.0 \times 10^{-3}$
Yankee 1	Pressurized Water	-2.2×10^{-3}

coefficient mainly applies to liquid moderated reactors. There are, of course, no voids in a gas-cooled fast (unmoderated) reactor system.

EFFECTS OF FISSION PRODUCTS ON REACTIVITY: POISONS

In the course of routine operation of a reactor, the fission fragments and their β^--decay daughter products accumulate in the fuel of the reactor. Several of these, such as ^{135}Xe and ^{149}Sm, have very large neutron absorption cross sections. Therefore, they behave the same as control rods in the six-factor formula: by decreasing the thermal utilization factor, they lower the k_{eff} and lower the reactivity. Thus, these fission products are known as poisons, since they poison or lower the value of k_{eff}. Any material which has a high thermal neutron cross section is called a poison; thus, control rods and chemical shims are poisons.

The effects of fission product poisoning were first noted in September, 1944, in operating with the B reactor at Hanford. This graphite moderated plutonium production reactor was beginning its initial power run and was to be run for a time at a low power level. After about an hour, the power level began to drop, and in four and one-half hours the B reactor had shut itself down completely. The scientists present feverishly looked for the cause of the shutdown.

Oddly enough, the power level began to increase a day later, and again the low step of the power run was tried. Again, after about an hour, the same decline in power began. Since there was no other explanation available, the scientists assumed that, as the reactor operated, a fission product with a high neutron cross section was produced, building up to a level sufficiently great to shut the reactor down. Once the reactor had been shut down, no more of the fission product poison would be formed. Then, as the radioactive fission product decayed away, the reactor could again be started. Careful study of the time dynamics revealed that the poison so produced was ^{135}Xe, a relatively high yield fission product

with a half-life of 9.2 hours. The thermal neutron capture cross section for ^{135}Xe is 2.6×10^6 barns, compared with the thermal fission cross section of 580 barns for ^{235}U.

Recall that the thermal utilization factor, f, for a non-poisoned (clean) reactor is defined by:

$$f = \frac{\Sigma_F}{\Sigma_F + \Sigma_M} \qquad \text{Eq. 12-7-3}$$

where Σ_F and Σ_M are the representative macroscopic cross sections for fission of everything except the fuel. For the poisoned reactor, the thermal utilization factor becomes smaller and is represented by f_p:

$$f_p = \frac{\Sigma_F}{\Sigma_F + \Sigma_M + \Sigma_p}. \qquad \text{Eq. 12-7-4}$$

In this equation, Σ_p is the macroscopic cross section for the poison. Note that Σ_p will change in proportion to the amount of poison in the core.

If f and f_p are placed in the four-factor formula for a nonpoisoned and poisoned core, Equations 12-7-5 and 12-7-6 can be derived as shown:

$$k_\infty = \varepsilon p f \nu = \varepsilon p \nu \frac{\Sigma_F}{\Sigma_F + \Sigma_M} = 1.000. \qquad \text{Eq. 12-7-5}$$

$$k_{\infty_p} = \varepsilon p f \nu = \varepsilon p \nu \frac{\Sigma_F}{\Sigma_F + \Sigma_M + \Sigma_p}. \qquad \text{Eq. 12-7-6}$$

The negative reactivity due to the poisons, ρ_p is defined by Equation 12-7-7. If Equations 12-7-5 and 12-7-6 are substituted into Equation 12-7-7, Equation 12-7-8 is developed:

$$\rho_p = \frac{k_\infty - k_{\infty_p}}{k_\infty}. \qquad \text{Eq. 12-7-7}$$

$$\rho_p = \frac{f - f_p}{f} = - \frac{\Sigma_p}{\Sigma_F + \Sigma_M}. \qquad \text{Eq. 12-7-8}$$

Solving Equation 12-7-5 for $(\Sigma_F + \Sigma_M)$ and substituting this solution into Equation 12-7-8 produces Equation 12-7-9, which provides an important way of expressing the reactivity of a poisoned reactor, ρ_p:

$$\rho_p = - \frac{\Sigma_p / \Sigma_F}{\varepsilon p \nu}. \qquad \text{Eq. 12-7-9}$$

Equation 12-7-9 is the primary equation that relates the macroscopic cross section due to poison and several other factors in the reactor, which can be easily estimated, in order to determine the negative reactivity due to the poison.

In the original consideration of both the six-factor formula and the time-dependent reactor, no consideration was given to these poisons because their concentration changes very slowly (on the order of many hours) compared to the rapid nature of the neutron density fluctuations.

As was mentioned, the two major poisons of interest are ^{135}Xe and ^{149}Sm. Table 12-7-3 summarizes the important

properties of each of these poisons. It should be noted

TABLE 12-7-3
PROPERTIES OF ^{135}Xe AND ^{149}Sm

	^{135}Xe	^{149}Sm
Independent Fission Product Yield-Direct Yield (atoms/^{235}U Fission)	0.00237	0.00000
Cumulative Fission Product Yield (atoms/^{235}U Fission)	0.0063	0.0107
Half-life	9.2 hours	Stable
(n,γ) cross-section, barns	3×10^6	5×10^4
Half-life of parent	6.7 hours	53 hours

that ^{149}Sm is stable compared to the 9.2 hour half-life of the radioactive ^{135}Xe while the absorption cross section of the latter (^{135}Xe) is 3×10^6 barns compared to the former's (^{149}Sm) 5×10^4 barns. As a result, the ^{149}Sm will be studied first because of its relative ease in quantification and more minor role its effect plays on reactivity.

The decay chain leading to ^{149}Sm is shown below:

$$\text{Fission} \rightarrow {}^{149}Nd \xrightarrow[2\ min]{\beta^-} {}^{149}Pm \xrightarrow[53\ hr]{\beta^-} {}^{149}Sm \text{ (stable)}.$$

Thus ^{149}Sm grows in the reactor fuel only by way of the β^--decay process; its stability indicates that the only way it can leave the core is via the ^{149}Sm (n,γ) ^{150}Sm capture

reaction, which is illustrated schematically in (Figure 12-7-2), and for which the cross section is 5 x 10^4 barns.

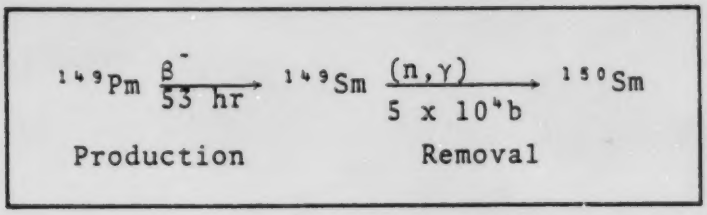

Figure 12-7-2. Path by which ^{149}Sm enters and leaves a reactor core.

After an infinite period of operation, the ^{149}Sm concentration in the reactor will reach an equilibrium concentration given by:

$$^{149}Sm_{\alpha} = \frac{\gamma_{149}\,\Sigma_f}{\sigma_{Sm}} \qquad \text{Eq. 12-7-10}$$

where

$^{149}Sm_{\alpha}$ = ^{149}Sm concentration after infinite operating time.

γ_{149} = fission yield of the A=149 chain (equal to 0.0107).

Σ_f = macroscopic fission cross section.

σ_{Sm} = cross section for the ^{149}Sm (n,γ) ^{150}Sm reaction (equal to 5 x 10^4 barns).

At equilibrium (or infinite time), the amount of ^{149}Sm

being produced is exactly equal to the amount being used to produce ^{150}Sm; thus, there is no gain or loss in the amount of poison. (The ^{150}Sm being produced does not count as a poison, for its cross section of 104 barns is very small compared to that of ^{149}Sm.)

Note: Student is advised that a detailed calculation using the above material is included in Appendix A.

Following shutdown of a reactor, no more ^{149}Sm can be converted to ^{150}Sm since there is no more flux; hence, the exit pathway is closed. However, the ^{149}Pm already produced continues to decay, producing more ^{149}Sm, according to:

$$^{149}Sm(t) = {}^{149}Sm_o + {}^{149}Pm_o\ (1 - e^{-\lambda t}) \qquad \text{Eq. 12-7-11}$$

where

$^{149}Sm(t)$ = concentration of ^{149}Sm present at time t after shutdown.

$^{149}Sm_o$ = concentration of ^{149}Sm at instant of shutdown.

$^{149}Pm_o$ = concentration of ^{149}Pm at the instant of shutdown.

λ = decay constant for ^{149}Pm.

As t approaches infinity, $^{149}Sm(t) = {}^{149}Sm_o + {}^{149}Pm_o$, since all the ^{149}Pm ultimately decays to ^{149}Sm.

When the reactor is subsequently started and taken to power, the ^{149}Sm will decrease to the equilibrium value associated with that power level. Prediction of this situation is essential for a good estimate of rod position at

which the reactor will be critical.

Since the $^{149}Pm_o$ varies with reactor flux before shutdown, $^{149}Sm(t)$ must also vary with the preshutdown flux (Figure 12-7-3).

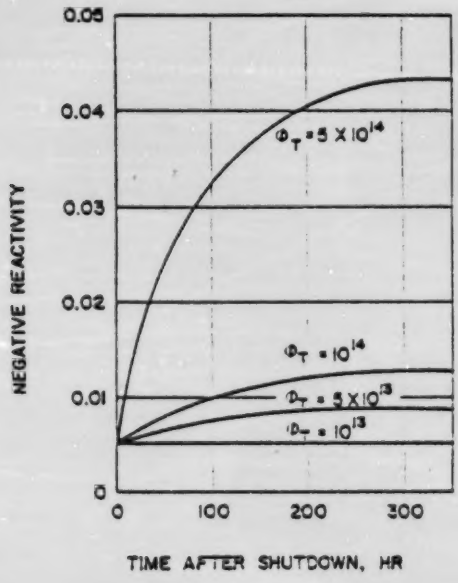

Figure 12-7-3.
Negative reactivity due to ^{149}Sm as a function of flux and time after shutdown.

From the standpoint of poisoning, ^{135}Xe is the most important fission product. This nuclide has a thermal neutron capture cross section of 3×10^6 barns and is formed both directly from fission and also from the decay of ^{135}I. The amount of ^{135}Xe in the core at any one time is in part a function of the ^{135}I decaying to ^{135}Xe plus the fission products going straight to ^{135}Xe. The poison can leave by two different routes: burnup or decay. Hence the amount of ^{135}Xe in the core is the solution to a complicated mathematical equation (a differential) relating flux, time, present ^{135}I concentration and the decay constants of ^{135}I and ^{135}Xe (Figure 12-7-4).

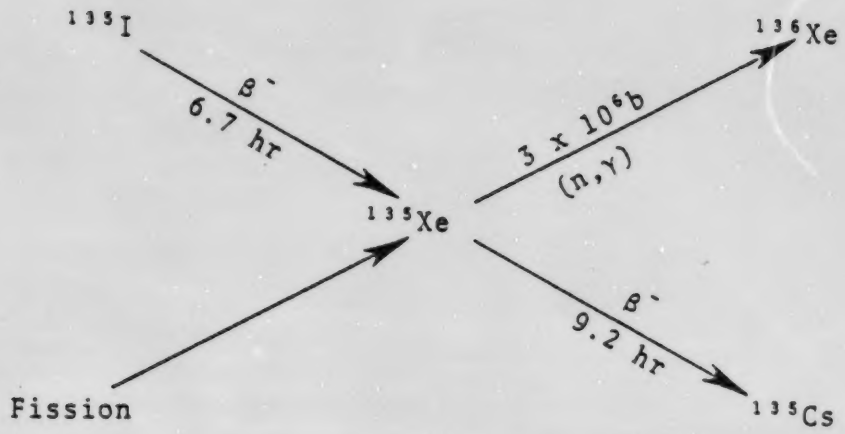

Figure 12-7-4. Paths by which ^{135}Xe can enter and leave reactor core.

After a reactor has been in operation for several days, both the ^{135}Xe and the ^{135}I buildup and maintain an equilibrium condition.

Note: A detailed calculation concerning the above material is included in Appendix B.

When a reactor is shut down, production of ^{135}Xe by fission ceases, although the nuclide continues to be produced by β^- decay of ^{135}I. Initially after shutdown, the amount of ^{135}Xe increases and reaches a maximum, after which it begins to decrease through radioactive decay:

$$^{135}I \xrightarrow[6.7\ min]{\beta^-} {}^{135}Xe \xrightarrow[9.2\ hour]{\beta^-} {}^{135}Cs. \qquad \text{Eq. 12-7-12}$$

The concentration of ^{135}Xe present at any time t after

shutdown, Xe(t), is a function of the concentrations of ^{135}Xe and ^{135}I present at the instant of shutdown and the radioactive decay constants of both nuclides. This can be calculated by a complex equation which, when combined with the macroscopic cross section for ^{135}Xe, shows that ρ_{Xe} is directly proportional to ϕ_T, the thermal neutron flux.

Buildup and decay of ^{135}Xe is a function of time after shutdown (Figure 12-7-5). Note that ρ_{Xe} increases (in nega-

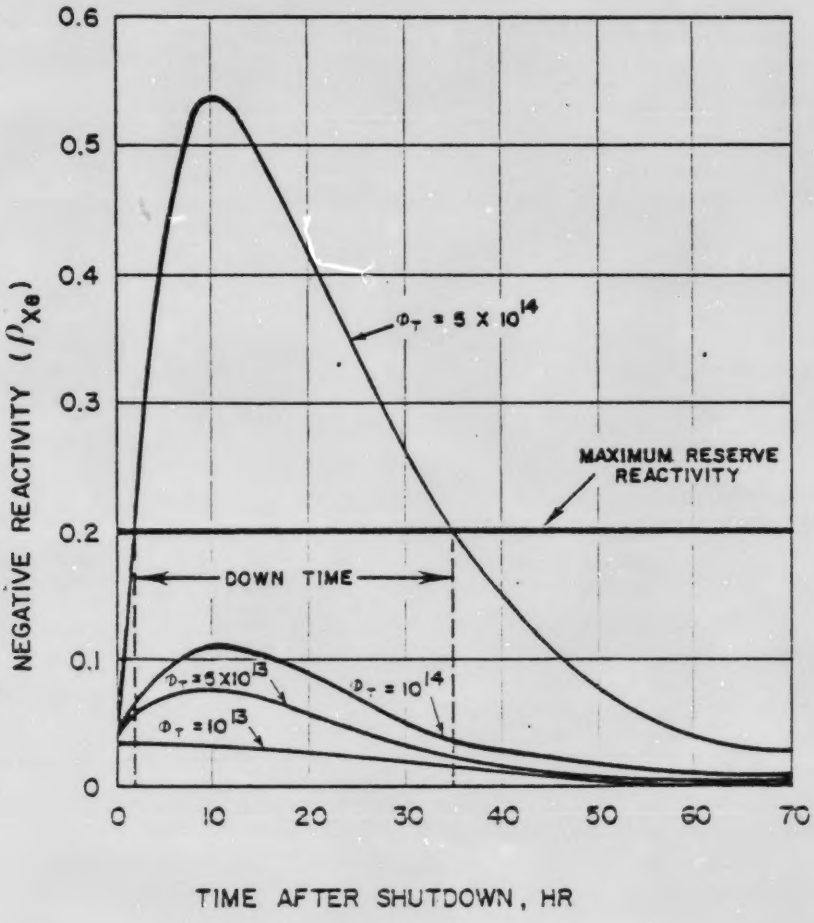

Figure 12-7-5. Negative reactivity due to ^{135}Xe following shutdown as a function of time after shutdown.

tive values) to a maximum value about 10.5 hours after shutdown, then decreasing with a half-life of 9.2 hours as the ^{135}Xe decays away.

The negative reactivity after shutdown due to ^{135}Xe is directly proportional to the flux; in a low flux facility this is of little importance, but in high flux reactors (such as power reactors) it may be very troublesome. In particular, if the positive reactivity achieved by complete removal of the control rods is less than the post-shutdown negative Xe reactivity, then the reactor will not be able to go critical. In this situation (Figure 12-7-5), the horizontal line across the graph at $\rho = 0.2$ represents the maximum positive reactivity achieved when all of the control rods are removed from the example core. For a flux, ϕ_T, of 5×10^{14} it can be seen that the reactor cannot be brought to a critical condition between about 3 hours and 35 hours. This period of time is known as the reactor deadtime. This time increases near the end of a fuel cycle when the available excess reactivity may be very small.

It is not necessary to completely shut down a reactor for the ^{135}Xe to increase over the equilibrium value. Any reduction in power causes an increase in the ^{135}Xe because less of the poison is being consumed due to burnup at the lower flux (Figure 12-7-6). Likewise, ^{135}Xe initially drops following an abrupt increase in power because of the increased burnup rate at the higher power.

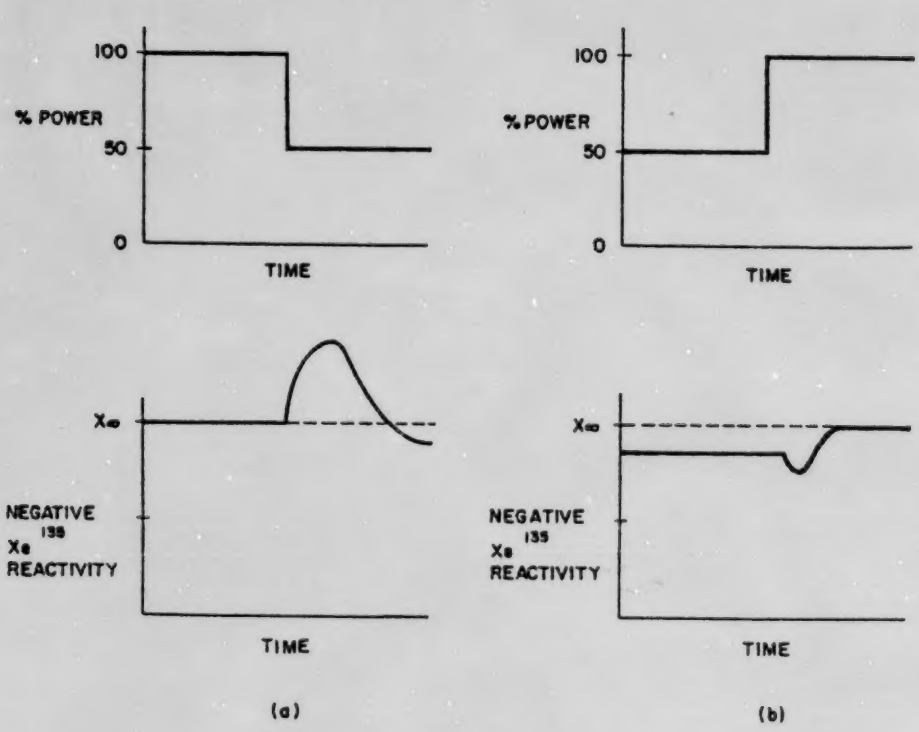

Figure 12-7-6. Changes in ^{135}Xe concentration following step changes in power, a) from full to half power and b) from half to full power.

EXERCISES

1. What are safety significances of temperature and void coefficients? Would you expect these to be as important as fission products in determining rapid changes in reactivity?
2. Show path by which ^{149}Sm is produced and how it leaves reactor core.
3. Given Figure 12-7-5 and a reactor with a flux of 1.0×10^{14} and a maximum reactivity of 0.3, would you expect

to be able to start it up 8.5 hours after shutdown?

4. Why does Sm come to an equilibrium rather slowly but, once into equilibrium, stay there?
5. Sketch various pathways for ^{135}Xe to enter and leave a reactor core.
6. Assume in an infinite reactor with $\varepsilon = p = 0.95$ and $\nu = 2.42$; calculate reactivity loss due to an equilibrium condition of ^{149}Sm; convert answer to dollars, assuming $\beta = 0.70$.
7. Assume same parameters as in exercise 6; calculate reactivity loss if the flux is a) 5×10^{14} and b) 5×10^{10} due to an equilibrium condition of ^{135}Xe.
8. Sketch curve of ^{135}Xe concentration following a sudden reduction in reactor power.
9. State and show why some reactors cannot become critical several hours following shutdown.

MATERIALS

Training reactor access.
Calculator.

EXPERIMENTAL MEASUREMENT OF THE TEMPERATURE COEFFICIENT OF A TRAINING REACTOR.

To accomplish this procedure, the reactor is assumed to be one in which the core excess can be easily measured as a function of coolant temperature. The easiest way to accomplish this task is to warm the coolant water up and then measure the core excess as the cooling system of the facility cools the water down. Of course, the experiment can be accomplished with the cooling water temperature increasing, but if this is done using the reactor to heat up the coolant, the effects of Xe poisoning may confuse the interpretation of the results:

1. As quickly as possible, use the reactor to heat the coolant to close to the maximum allowed temperature. If the cooling system has an auxiliary heater unit, by all means use this instead of the reactor. (A good exercise while waiting for the water to warm up is to review Module 12-3, "Fission Rate and Reactor Power," and calculate the length of time required for the reactor to deliver the necessary heat to the pool to raise the water the necessary amount.)
2. Turn the cooling system on and measure the core excess at a low power level.
3. Leave the cooling system on and measure the core excess as the pool water cools down. If the cooling rate is rather slow, shut the reactor down between readings. Record the data on Data Table 12-7-1.
4. When the pool water has returned to its ambient temperature, analyze the data as directed at the bottom of Data Table 12-7-1.

DETERMINE THE AFFECT OF ^{135}Xe POISONING ON A REACTOR

This experiment can only be done successfully if a high flux reactor which has to run near equilibrium ^{135}Xe conditions is available. Following this running time, the reactor is to be shut down and must be available for the experiment for at least the next 30 hours.

The purpose of this experiment is to observe the post-shutdown ^{135}Xe poisoning of a reactor. A curve similar to Figure 12-7-5 is to be constructed.

Following shutdown of the reactor, bring the reactor back to a low power level and measure the core excess. Record the data on Data Table 12-7-2. Following the times outlined on that Data Table, bring the reactor back up in power to the same low power level and measure the core excess. Plot the data (core against time) on linear graph paper.

DATA TABLE 12-7-1

PRE-EXPERIMENTAL DATA

Ambient Cool Water Temperature _____

β _____

Power Level at Which Core Excess is to be Measured _____

DATA FOLLOWING HEATUP OF POOL WATER

Time	Water Temperature	Core Excess, $

DIRECTIONS FOR OBTAINING THE TEMPERATURE COEFFICIENT

1. Plot the water temperature versus core excess on linear graph paper.
2. Determine the slope of the best line fit through the data points. Remember that, since the water temperature was going down while the core excess should have been increasing, the slope will have a negative sign.

The units will be dollars per degree or ρ per degree.

3. Multiply the $/degree to obtain the temperature coefficient in reactivity, ρ, per degree.

DATA TABLE 12-7-2

Date and time of shutdown: _____

Power at which core excess will be measured: _____

Time	Desired Hours Since Shutdown	Real Hours Since Shutdown	Core Excess, $

REFERENCE MATERIALS

Bennet, D.J. The Elements of Nuclear Power. New York: John Wiley & Sons, 1973.

Glasstone, Samuel and Sesonske, Alexander. Nuclear Reactor Engineering. New York: D. Van Nostrand, 1967.

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_____. Introduction to Nuclear Reactor Theory. Reading, MA: Addison-Wesley Publishing Co., 1966.

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Pedersen, Knud B.; Sasscer, Donald S.; Plaza, Herbierto; Gileadi, Aviva E.; Brown-Campos, Richard; and Holyoak, Robert H. Applied Nuclear Power Engineering for Practicing Engineers. Boston: Cahners Books, 1972.

APPENDIX A

If Ni $\sigma i = \Sigma i$, then $Sm_{\alpha}\ \sigma_{Sm} = \Sigma_{Sm}$, and, by substitution in Equation 12-7-10, we have:

$$\Sigma_{Sm} = \gamma_{149}\ \Sigma_f. \qquad \text{Eq. 12-7-13}$$

Since Σ_{Sm} in Equation 12-7-13 is equivalent to Σ_p in Equation 12-7-9, the reactivity decrease due to ^{149}Sm poisoning, ρ_{Sm}, is given by:

$$\rho_{Sm} = -\frac{\Sigma_{Sm}/\Sigma_f}{\varepsilon p \nu} = -\frac{\gamma_{149}}{\varepsilon p \nu} \qquad \text{Eq. 12-7-14}$$

or

$$-\rho_{Sm} = \frac{\gamma_{149}}{\varepsilon p \nu}.$$

Note that ρ_{Sm} is independent of reactor flux or power.
If it is assumed that $\varepsilon = p = 1.00$ and $\nu = 2.42$ with $\beta_{eff} = 0.0075$, then ρ_{Sm} can be calculated as:

$$\rho_{Sm} = -\frac{\gamma_{149}}{\varepsilon p \nu} = \frac{0.01.7}{1 \times 1 \times 2.42}$$

$$= -0.0044 = -0.44\%$$

or in dollars,

$$\$ = \frac{\rho_{Sm}}{\beta_{eff}}$$

$$= \frac{-\ 0.0044}{0.0075} = 0.59.$$

This calculation is valid only for the equilibrium condition. Because of the stability of ^{149}Sm, 99.9% of equilibrium is reached 530 hours (22.1 days) after starting with fresh fuel.

APPENDIX B

The number of ^{135}Xe atoms present is given by Equation 12-7-15 in which the γ_i's are the fission yields (atoms per fission) for I and Xe, and ϕ_T is the neutron flux. Thus, the macroscopic Xe cross section, Σ_{Xe}, is $Xe_\infty \sigma_{Xe}$ and Equation 12-7-15 is transformed into Equation 12-7-16 by substitution.

$$Xe_\infty = \frac{(\gamma_I + \gamma_{Xe})\ \Sigma_f\ \phi_T}{\lambda_{Xe} + \sigma_{Xe}\ \phi_T} \qquad \text{Eq. 12-7-15}$$

$$\Sigma_{Xe} = \frac{(\gamma_I + \gamma_{Xe})\ \Sigma_f\ \sigma_{Xe}\ \phi_T}{\lambda_{Xe} + \sigma_{Xe}\ \phi_T}. \qquad \text{Eq. 12-7-16}$$

If the ratio of λ_{Xe} to σ_{Xe} is set equal to ϕ_{Xe}, Equation 12-7-16 simplifies to:

$$\Sigma_{Xe} = \frac{(\gamma_I + \gamma_{Xe})\ \Sigma_f\ \phi_T}{\phi_{Xe} + \phi_T}. \qquad \text{Eq. 12-7-17}$$

In Equation 12-7-17, ϕ_{Xe} has units of atoms/cm^2-s, and its calculated value is 8×10^{12}.

Equation 12-7-17 can be combined with Equation 12-7-9 to give the reactivity associated with ^{135}Xe poisoning, ρ_{Xe}, as shown in Equation 12-7-17:

$$\rho_{Xe} = - \frac{(\gamma_I + \gamma_{Xe})\ \phi_T}{\varepsilon p \nu\ (\phi_{Xe} + \phi_T)} . \qquad \text{Eq. 12-7-18}$$

If the values for γ_I, ϕ_{Xe}, and γ_{Xe} are put into Equation 12-7-18 and it is assumed that $\varepsilon = p = 1.000$ and $\gamma = 2.42$, then the only unknown variable remaining is the thermal neutron flux. Since $\phi_T >> \phi_{Xe}$, $(\phi_{Xe} + \phi_T) \approx \phi_T$, and Equation 12-7-18 reduces to:

$$\rho_{Xe} = - \frac{(\gamma_I + \gamma_{Xe})}{\varepsilon p \nu} \qquad \text{Eq. 12-7-19}$$

since the two ϕ_T's cancel out, when the various constants are put in:

$$\rho_{Xe} = - 0.0274 = - 2.74\% \qquad \text{Eq. 12-7-20}$$

or, in units of dollars, with $\beta_{eff} = 0.0075$,

$$\$ = \frac{\rho_{Xe}}{\beta_{eff}} \qquad \text{Eq. 12-7-21}$$

$$= - \frac{- 0.0274}{0.0075} = - \$3.65$$

or a \$3.65 reduction in reactivity. Hence, an additional

$3.65 worth of reactivity is required to override the ^{135}Xe poisoning at equilibrium.

It is also possible that $\phi_{Xe} >> \phi_T$. In this case, Equation 12-7-19 reduces to:

$$\rho_{Xe} = \frac{- 0.0274\ \phi_T}{8 \times 10^{12}} \qquad \text{Eq. 12-7-22}$$

$$= - 3.42 \times 10^{-15}\ \phi_T$$

1. Explain general effect of fission products on reactivity:

2. Write nuclear reaction equations for production and removal of ^{135}Xe:

3. State (in terms used in this module) how ^{135}Xe affects reactivity:

4. Given an equation for the rate of change for ^{135}Xe, define and give significance of each term:

5. Plot ^{135}Xe concentrations-versus-time for the following conditions:

   Xe free.

   Startup immediately following full power at equilibrium Xe conditions.

   Startup at peak Xe condition.

6. Briefly explain (in terms used in this module) why some power reactors are not able to start up a few hours after shutdown:

7. List effects of ^{149}Sm on reactivity; give appropriate comparisons with ^{135}Xe:

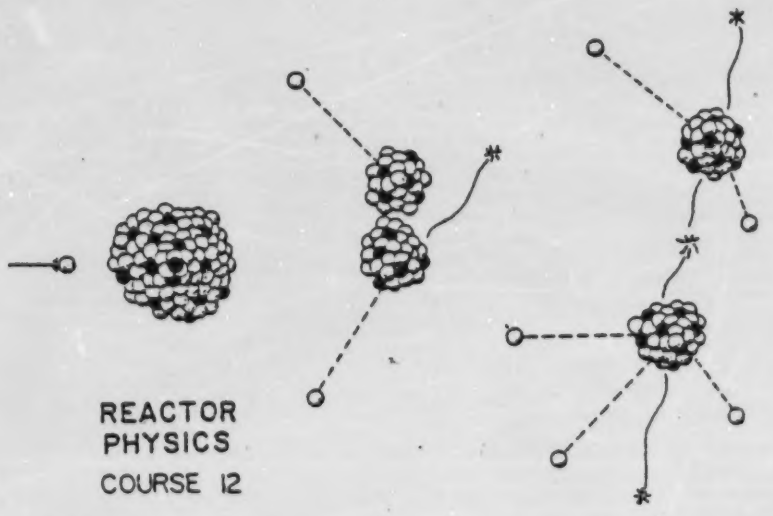
8. Describe (in terms used in this module) general effects of temperature on reactivity:

9. Briefly explain how voids affect reactivity; give at least three typical void coefficients for existing nuclear reactors:

NUCLEAR TECHNOLOGY

MODULE 12-8 REACTOR OPERATING CHARACTERISTICS. -

RONALD L. KATHREN AND LARRY B. CHURCH



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COURSE MODULES

COURSE 12 REACTOR PHYSICS

MODULE 12-1 Neutron Physics

MODULE 12-2 Cross Sections and Reactions

MODULE 12-3 Fission Rate and Reactor Power

MODULE 12-4 Time Independent Nuclear Theory

MODULE 12-5 Time Dependent Nuclear Theory

MODULE 12-6 Reactivity - Fuel Moderator and Control Systems

MODULE 12-7 Internal Effects on Reactivity

MODULE 12-8 Reactor Operating Characteristics

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INTRODUCTION

Although all reactors are constructed and operated according to the same general principles, each has different and sometimes specific operating characteristics. These characteristics may serve as a means of classifying all reactors of that type. In general, however, reactors are classified in one or more of the following ways:

By physical design.
By coolant.
By moderator.
By use.

In this module, reactor characteristics are examined by considering, primarily, three basic types of power reactors in use in the United States:

Pressurized water reactor (PWR).
Boiling water reactor (BWR).
Gas-cooled reactor (GCR).

Other reactor designs will be considered as well, including heavy-water moderated, fast breeder, sodium cooled and certain unique reactors used in specific research and training applications.

MODULE PREREQUISITES

The student should have completed all previous modules in this Course.

OBJECTIVES

Upon completion of this module, the student should be able to:

1. Differentiate between gas-cooled and light-water cooled reactors on the basis of design features.
2. Discuss basic operating parameters of a PWR, a BWR and a GCR.
3. Calculate conversion efficiency of a typical steam-electric system.
4. Calculate conversion ratio for operating reactors.
5. Explain three advantages of heavy-water moderation.
6. Explain how a fast reactor works; give basic equations for breeding.
7. Differentiate among breeding ratio, conversion ration, breeding gain, conversion efficiency.
8. Discuss effects of voids on BWR, PWR and GCR operation.

GENERAL OVERVIEW OF POWER REACTORS

Pressurized water reactors, PWR's, are the most common power reactors in use in the United States and the world. The three principal American manufacturers of PWR's are Babcock and Wilcox, Combustion Engineering and Westinghouse. Although there are important differences in the specific design and operating conditions, all three manufacturers' reactors are similar, hence PWR's are considered as a group.

Several foreign companies, including Kraftwerk Union Ag (West Germany), Framatome-Creusot-Loire (France), Atomenergo Export (Russia), Mitsubishi Heavy Industries (Japan) and Elettronucleare Italeane (Italy) also supply PWR's, which also have operating characteristics similar to those of American manufacturers.

PWR's are all light-water reactors (LWR's) - they use ordinary water as coolant-moderator. Some PWR's use heavy water as moderators (in other countries, e.g., Canada's Candu reactor), but there are none in the United States. (Heavy-water reactors are considered later in this module.)

PWR is one of two important subcategories of LWR's, the other being the boiling water reactor (BWR). Nearly all power reactors in the U.S. are LWR's; of the 211 American power reactors operating, under construction or on order as of 1977, 140 (approximately two-thirds) are PWR's. Table 12-8-1 gives a breakdown of American power reactors by type (as of January 1, 1977).

Total electrical power to be generated by these 211 reactors is in excess of 206 gigawatts (1 gigawatt = 1000 megawatts = 1,000,000 kilowatts), or nearly half of the

TABLE 12-8-1

AMERICAN POWER REACTORS ACCORDING TO TYPE
(JANUARY 1, 1977)

REACTOR TYPE	NO. OPERATING	% OF TOTAL	OPERATING ORDERED PLANNED	% OF TOTAL
Pressurized Water	34	55	140	66
Boiling Water	25	40	68	32
Gas Cooled	1	--	1	--
Graphite Moderated	1	5	1	2
Fast Breeder	1	--	1	--
TOTALS	62	100	211	100

projected electrical power need of the U.S. in the year 2000.

The popularity of light-water reactors is obvious. Water is both a good coolant and moderator, plentiful and inexpensive, and easily purified. Also, it has well understood properties. Indeed, generation of electricity by steam was well developed long before the discovery of fission processes. Burning of fossil fuels such as coal, oil or wood waste ("hog fuel") have been common ways of generating heat to produce steam to drive a turbine for a long time.

An extremely important concept, and one which determines many design and operating characteristics, is the efficiency of a heat engine in converting heat into electricity. Both heat and electricity are forms of energy. According to the laws of thermodynamics, energy can be trans-

ferred from one form to another; however, in so doing, some otherwise useful energy is always lost:

$$\% \text{ efficiency} = \frac{\text{output}}{\text{input}} \times 100$$

$$= \frac{\text{high temperature - low temperature}}{\text{high temperature}} = 100$$

or, using the symbols T_1 and T_2 to represent low and high temperature, respectively,

$$\% \text{ eff} = \frac{T_2 - T_1}{T_2} \times 100.$$

Equation 12-8-1
(Equation = Eq.)

All temperatures are expressed as absolute temperature, either in degrees Kelvin or Rankine. Equation 12-8-1 provides a means of calculating theortecial maximum efficiency for converting heat (in the case of a power reactor, via steam) into mechanical energy to drive a turbine and produce electricity. This efficiency is called Carnot, after the French engineer, Sade Carnot.

Two important points must be stressed with regard to Equation 12-8-1. First, it provides a theoretical upper limit; in practice, actual efficiency is much less because of losses attributable to:

Friction.

Finite size (and hence heat loss through walls by conduction and radiation, etc.)

Energy to heat components.

In addition, temperatures used in Equation 12-8-1 are in absolute degrees; thus, to obtain 100% efficiency, it would be necessary for T_1 to be 0, or for cooling to lower the temperature to absolute zero, an obvious engineering impossibility.

The Carnot efficiency equation also shows reasons why light-water cooled reactors may, from some standpoints, be undesirable. Steam temperatures have relatively low upper limits, and higher temperatures would permit a greater amount of cooling or a larger ΔT (temperature difference), thus increasing efficiency of heat conversion to electricity. Hence, gas and liquid-metal-cooled reactors have had an obvious appeal because of their potentially higher temperature difference.

Unlike the U.S., which features light-water cooled reactor technology, other countries (notably Great Britain) have proceeded with gas-cooled reactors (GCR's). This design permits higher operating temperatures, and hence higher conversion efficiencies. By 1977, there were 31 gas-cooled power reactors operational in Great Britain with a total capacity of 6.8 gigawatts of electricity. Gas-cooled reactors are generally of smaller size than light-water cooled reactors; the largest British reactor has a rating of 625 MWe compared with a typical 1100 MWe for a PWR. In physical size, light-water reactors are even larger because of their lower conversion efficiency. Thus, a large GCR may have a power level of about 1600 MWth, or slightly less than half that of a very large light-water cooled reactor.

As of early 1977, the largest power reactor in the world was the Bibli's A (Worms, West Germany). This PWR, with a rated net capacity of 1147 MWe, was built by Simms Electric. Bibli's A began commercial operation in June, 1974.

The largest American power reactor at that time was the Trojan Nuclear Plant near Portland, Oregon. This PWR, with a rated electrical output of 1130 MW, began commercial operation in December, 1975. Significant power increases are not anticipated with future reactors; the largest planned units are all PWR's with projected ratings of ~ 1300 MWe to be built in France, West Germany and Luxembourg. The largest planned units in the U.S. are two 1288 MWe BWR's, the Skagit 1 and 2 plants, near Sedro Wooley, Washington, scheduled for completion in the mid 1980's.

Another important class of power reactors, pioneered by the Canadians, are the pressurized heavy-water moderated and cooled reactors supplied by Atomic Energy of Canada, Limited. Known as the CANDU (pronounced "can do") reactors, the largest of these is the Bruce 2 plant near Tiverton, Ontario, with a rated power level of 746 MWe. Heavy-water moderated reactors are also operational in India, Argentina, Czechoslovakia and West Germany.

As of 1977, 31 countries were known to have had operating power reactors or to have announced plans to construct one or more (Table 12-8-2). Note that light-water reactors account for about 63% of the total operating power reactors as of 1977, with gas-cooled comprising another 30%. Virtually all the small, gas-cooled reactors are located in Great Britain and France. The heavy-water moderated reactors are mostly in Canada, and four of the light-water/graphite moderated reactors are Russian. The fifth is the 800-MWe N reactor at Hanford, Washington, designed as a dual purpose power and plutonium production reactor.

Although several breeder reactors are under construction, as of 1977, only two were in operation: the 233-MWe Phenix near Marcoule, France, and the 350-MWe Russian

TABLE 12-8-2

WORLD POWER REACTOR SUMMARY

Countries With Operating Power Reactors:	19	
Countries With Power Reactors Planned:	12	
Types Of Reactors Operating:	-	
Type	Number	% Of Total
Pressurized Water	63	37
Boiling Water	45	26
Gas Cooled	51*	30
Heavy Water Moderated	12*	7
Water Cooled, Graphite Moderated	5	3
Breeder	2	1
TOTAL	171*	100*

* Do not add up since hybrid reactors were placed in more than one category.

BN-350 plant. The BN-350 began commercial operation in June, 1972, and the Phenix the following year.

It is interesting to note that all the world's power reactors can be lumped into just six classes, with LWR's dominating. The reasons for this domination will become apparent as the specific characteristics are presented.

CHARACTERISTICS OF A PWR

The popularity of the PWR, in large measure, has resulted from its evolution from the U.S. Navy reactor program. A PWR powered the first nuclear submarine - the USS Nautilus - when it began its sea trials in January, 1955. Three years later, the first civilian application became a reality when the Shippingport Atomic Power Station began full power operation at 60 MWe. This reactor is of special interest, not only from a historical standpoint, but because it used a special type of fuel loading, known as "seed-and-blanket."

In the seed-and-blanket system, 32 flat-plate assemblies (known as seed assemblies) of enriched uranium are arranged in a hollow square array. Inside and outside these seed assemblies are 113 subassemblies of natural uranium rods - the blanket assemblies. More than 90% of the active fuel - the ^{235}U - is contained in the seed assemblies, but criticality cannot be sustained by the seed assemblies alone. To increase k_{eff} (effective multiplication constant) to ≥ 1.000, reflecting capability plus a tiny addition to k_{eff} from the ^{235}U in the blanket are necessary. Thus, there are certain safety advantages as well as advantages of uniform power distribution across the loaded core. There are also disadvantages, including low conversion efficiency and difficulty of heat removal, since most heat is generated in the seed subassembly.

The seed-and-blanket concept has given way to more modern core designs, using from 2 to 3% enriched fuel rods tied together in bundles to form fuel assemblies. A typical assembly in a large PWR may be 10 to 12 feet long and contain about 150 rods. And each core may contain 200 or

so assemblies; hence, a large PWR may contain about 40,000 individual fuel rods, each weighing about 4.5 pounds and containing numerous fuel pellets. The large Trojan Nuclear Power Plant contains 50,952 fuel rods, each 12 feet long and 3/8 inch in diameter, in 193 fuel assemblies, each comprised of 264 rods.

In a typical PWR there are three coolant loops (Figure 12-8-1). The primary coolant, which cools the reactor core,

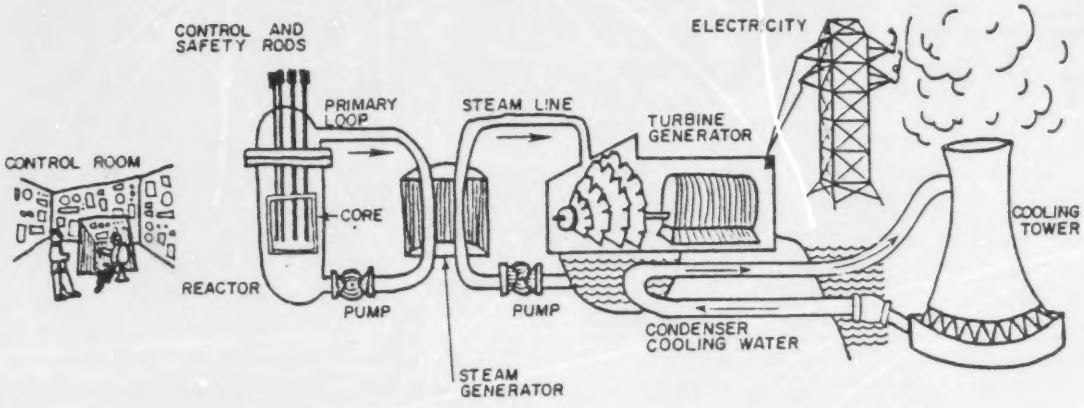

Figure 12-8-1. Pressurized water power reactor.

is kept under pressure and hence not allowed to boil. At a pressure of 2250 psi where most large power reactors operate, water does not boil even at temperatures of several hundred degrees. Thus, the primary coolant of a PWR may be at about 600°F and at a pressure of 2000 psi where the boiling point of water is 636°F (Figure 12-8-2). Some of the energy in this primary coolant is removed in the steam generators where it is used to boil water in the secondary

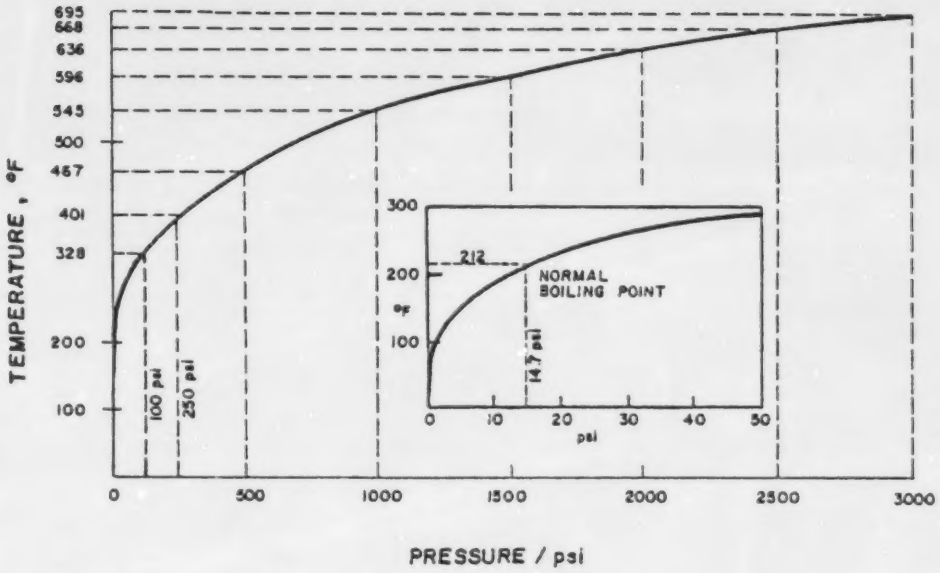

Figure 12-8-2. Relationship of boiling temperature to pressure for water.

loop and create steam; the steam pressure in this loop is on the order of 600 to 1000 psi and is used to drive a turbine, producing electricity and cooling itself in the process. The third loop is used to cool the condenser and the turbine.

Because of the pressure in the primary loop, a PWR can be made fairly small and with a high power density. Thus, they are desirable in applications such as submarines where space and weight must of necessity be limited. In addition, use of a single, inexpensive substance - water - for the important vital reactor functions of coolant, moderator and reflector is also advantageous. The PWR must use enriched fuel, although the enrichment need not be high; a typical 1000-MWe commercial PWR will use about 90 tons of 2 to 3% enriched uranium.

A PWR is inherently quite stable and has a relatively high negative temperature coefficient. A typical value is on the order of 10^{-4} $\Delta k/°F$. The PWR uses a closely packed core lattice, with a water-to-uranium volume ratio of about 3. This provides a relatively high core power density, or about 20 kW per gallon of water in the core. Average thermal neutron flux levels in a large PWR are on the order of 2×10^{13} $n/cm^2\text{-}s$ with maximum available k_{eff} ranging from about 1.20 for a cold, clean core down to about 1.07 for a hot core at full power with Xe and Sm at equilibrium.

Fuel burnup is an important consideration in any power reactor and, by careful fuel design and management, burnup rates on the order of 15,000 to 20,000 megawatt days per metric ton, tonne, (MWd/t) of fuel may be achieved. Although there are three separate cooling systems and hence an efficiency loss through conversion, a total conversion efficiency of about 34% can be obtained. Thus, a 1000-MWe modern reactor will have a gross rating of almost 3000 MWth; conversion efficiencies in older reactors were significantly lower, down to about 26 to 28%. (MWe = megawatts electric, MWth = megawatts thermal.)

Although PWR's use enriched uranium, the actual enrichment is low and thus an appreciable amount of plutonium will be produced. Since plutonium is a highly valuable material as a potential future fuel, a high conversion ratio is desirable; the conversion ratio, CR, is defined as the number of ^{239}Pu atoms produced for each ^{235}U atom fissioned. Typically, a PWR has a conversion ratio of about 0.5 to 0.75; hence, a 1000-MWe PWR, operating at a typical capacity factor of 0.70 produces about 600 to 800 pounds of ^{239}Pu per year:

$$CR = \frac{\sigma_c}{r(\sigma_f + \sigma_a)} + \Sigma P(1-p)n$$

Eq. 12-8-2

where

σ_c = capture cross section for ^{238}U.

σ_f = fission cross section for ^{235}U.

σ_a = absorption cross section for ^{235}U.

r = ratio of ^{235}U to ^{238}U in fuel.

Σ = fast fission factor.

P = nonleakage probability.

p = density of fuel-moderator.

n = neutrons produced per neutron absorbed by ^{235}U.

Note that, as reactor core parameters change, so will conversion ratio.

As has been mentioned, heavy water is used as moderator in the Canadian CANDU (Canadian Deuterium Uranium), which uses pressurized water in channels through the reactor core to achieve cooling. The advantage of heavy water is that it has superior moderating properties over light water, and permits the reactor to be fueled with natural uranium - a distinct economic advantage is gained, for the fuel enrichment process is quite expensive.

Note that the thermal neutron cross section for heavy water, D_2O, is only 1.1 millibarns compared with 660 millibarns for ordinary light water, indicating that heavy water is far less competitive for thermal neutrons.

An important advantage of heavy-water moderated reactors is their high conversion ratio which can approach unity in actual operation. This is in large measure due to the low enrichment (0.7%) of the natural uranium fuel, as can be seen from Equation 12-8-2 above. Note that this extends the life of the fuel greatly, since the ^{239}Pu produced is also fissile and hence is fissioned as well. Theoretically, a conversion ratio of 1.0 would allow for unlimited fuel lifetime, but production of fission-product poisons, of course, changes reactivity value of the rods, ultimately offsetting any gain.

An important operating characteristic typical of all PWR's is their inherent stability and ease of control, which is in large measure gained by the lack of voids as would be produced if the water were permitted to boil. Moreover, the liquid water state permits use of soluble poisons such as boric acid for chemical shimming with relative ease. PWR void coefficients are very small and essentially trivial in some reactors.

CHARACTERISTICS OF BOILING WATER REACTORS

Boiling water reactors (BWR's) offer an attractive alternative to PWR's because heat removal is far better if water is allowed to boil. If the water in the primary system is converted to steam, it can be used directly to drive the turbine-generator, thus eliminating the need for steam generators and indeed the entire secondary system of a PWR. The improved heat removal capabilities of water

permitted to boil also provide an additional advantage; that is, less primary coolant is required to obtain the required cooling. Thus, many expensive components, including pumps, demineralizers and even reactor vessels can be sized much smaller than for a PWR of the same rated power level.

Although experiments at the National Reactor Testing Station (NRTS) in Idaho (in 1953) showed that water could be safely boiled in a reactor, there were still serious doubts in the minds of many reactor physicists and engineers regarding the inherent fluctuations of power from void formation and collapse in the BWR. Steam, being of lower density than water, is a poorer moderator and tends to reduce reactivity. This difficulty is offset to some extent by the negative temperature coefficient; however, when a void collapses, a sudden rise in reactivity tends to lead to instability and control problems.

During 1953 and 1954, a series of control and safety tests were performed at NRTS on the BORAX-I (Boiling Reactor Experiment) reactor, which showed that steam formation in a reactor core reduces reactivity and causes the reactor to become self-regulating under reasonable operating conditions. BORAX-I was deliberately allowed to go out of control by adding successively greater amounts of reactivity until in July, 1954, a power excursion from an excess reactivity of 4% destroyed the reactor as expected and without serious consequences. These experiments have proven the validity of the boiling water concept for power reactors and in June, 1955, the first electric power generation by a boiling water reactor took place. The first practical BWR power plant prototype was the Experimental Boiling Water Reactor (EBWR) built in 1952. EBWR used both natural

uranium and fuel elements enriched to 1.44%.

A major advantage, already mentioned, of the BWR is its greatly simpler design. Additionally, BWR's operate at much lower pressures and hence require a thinner pressure vessel than PWR's. Typically, the primary coolant pressure in a BWR is about 1000 psi; at this pressure, water boils and forms steam at about 545°F (Figure 12-8-2).

Core designs of BWR's are similar to PWR's. Fuel rods are 10 to 12 feet in length with 50 to 65 rods making up one assembly. A large BWR core may contain as many as 764 assemblies and nearly 40,000 rods per core. Rods are slightly larger in diameter than PWR rods (about 0.5 inch compared with 0.374 inch) and the core may, therefore, contain about twice as much UO_2 fuel material - nearly 200 tons - as a PWR of comparable size. Thus, as might be expected, the core power density is also lower, about half that of a comparably rated PWR or about 150 kW per gallon. The limitation on power density is imposed by boiling; only about 3 to 5% (by weight) of the coolant passing through the core can be converted to steam per pass through the boiler. This is actually about 30 to 40 percent of the coolant volume (since steam has a lower density than water), and obviously it is more difficult to achieve a self-sustaining chain reaction with such a great percentage of the moderator outside the core. Therefore, a greater amount of fuel is needed to make up the reactivity loss from this absence of moderator. Fuel exposure, however, is comparable to that of PWR's, running about 15,000 MWd/t; and conversion ratios are thus similar to those of PWR's.

Because the voids and bubbles produced tie up reactivity, BWR's require more control rods than PWR's. Unlike a PWR, which tends to follow turbine demand (increase power output

when required by turbine), a BWR tends to have a negative power demand coefficient reactivity. This tendency results from the fall in steam pressure that occurs when turbine demand increases. Reduction of pressure in the reactor leads to formation of larger bubbles (voids) and reduction of moderation and reactivity, ultimately shutting down the reactor. This problem, which can be eliminated by careful design considerations, suggests that BWR's may become unstable during low-pressure, high-power operation. This stability failure, however, is on the side of safety, for the unit tends to turn itself off.

An important characteristic, at least of early BWR's, was the relatively high release of radioactive gases stripped from the steam in the primary system. In a PWR, these fission gases, which diffuse through the cladding, are taken out of the liquid coolant and held under pressure in radioactive gas waste storage tanks for several months prior to release, thus permitting radioactive decay to reduce radioactivity to acceptably low levels. In a BWR, steam is condensed after passing through the turbine, and condensate is passed through an air ejector to remove dissolved gases, which are discharged after a 30-minute holdup time. Gas removal is more critical in BWR's, which are required to maintain water quality and optimum thermodynamic performance. The gases may be held up in tanks to allow for decay and then passed through charcoal or other filters to further reduce radioactivity. Although modern BWR's discharge significantly greater quantities of radioactive gases than PWR's, levels are well within those considered acceptable by regulatory and standards-setting bodies.

Unlike PWR's, the tritium (^{3}H) produced in BWR's is minimal. In both types of reactors, about 1 Ci of ^{3}H is

produced each year for each 60 MWth; thus a 1000-MWe (or 3000-MWth) plant produces about 50 Ci of tritium annually. About 80% of this tritium is produced by ternary fission, with the remainder resulting from neutron capture by deuterium, normally present in a ratio of 1 to 6250 parts of ^{1}H in ordinary water.

In addition to this 50 Ci, a 1000-MWe PWR produces about 600 Ci from the $^{10}B(n,^{3}H)^{7}Be$ and other reactions from the boron used as a chemical shim in the coolant. This tritium must be removed and disposed of from time to time, generally by liquid release to the environment. From the standpoint of ^{3}H releases, BWR's are clearly superior to PWR's.

As with PWR's, the thermal efficiency of a BWR is in the region of 30 to 34%. Even early BWR's achieved high efficiencies during electrical power production; for example, the Big Rock Point plant, which began operation in 1962, has a 31% conversion efficiency (75 MWe from 240 MWth) and Dresden 1, an even earlier plant, has been able to achieve 27.5%.

GAS-COOLED AND OTHER REACTORS

For more than 20 years, the British have successfully used gas-cooled reactors (GCR's), which have certain inherent advantages and control differences with regard to LWR's. The original British reactors of the Calder Hall type are fueled with natural uranium, moderated with graphite (carbon) and cooled with carbon dioxide. Later models, advanced gas-cooled reactors, use 1.2% enriched uranium.

In the U.S., efforts have been directed toward production of a high-temperature gas-cooled reactor (HTGCR). The first of these was the Peach Bottom 1 demonstration plant, which began commercial operation in 1966, but is no longer operational. Only a single HTGCR - the Fort St. Vrain Plant near Denver - is in operation in the U.S., and the American supplier (General Atomics) ceased its sales and production efforts in 1975.

HTGCR's are fueled with high uranium enrichments (13.8% for Peach Bottom 1) and cooled with helium under pressure. Because of their higher temperatures, conversion efficiencies of these reactors are higher than the LWR's, ranging to about 40%. Because of the high enrichment, cores are quite small. Core power densities are very low, however, because of the large volumes of gas passed through the reactor. Fuel exposure is comparable to LWR's. A HTGCR, in addition to being smaller, will have only about 10% as many fuel elements as a LWR, and these will be smaller and more easily handled.

GCR's generally rely heavily on automatic controls since they do not have the inherent control features of LWR's, associated with moderator density changes. Inherently, they are unlikely to be seriously affected by a loss of coolant, and the use of a gas (such as CO_2 or He) eliminates all possibility of a steam explosion. In general, conversion ratios are quite high despite the high fuel enrichment used.

Many other reactor systems have been proposed and experimentally developed, including sodium cooled, organic moderated and homogeneous types. In a homogeneous system, fuel and moderator are mixed together to provide evenness of moderation. This evenness allows ease of control, even

with high power density and small critical mass (core size). In addition, homogeneous systems inherently have a high negative temperature coefficient and a positive power demand coefficient; however, above about 500°F, the fuel solution may separate into two fractions, making control difficult and complex.

Organic moderated and/or cooled reactors use organic compounds such as diphenyl ($C_{12}H_{10}$), which boil at fairly high temperatures, thus permitting good conversion efficiencies. Although inherently safe, such reactors have a great deal of excess reactivity. A major drawback is decomposition of the organic moderator used.

Fast reactors are those in which neutrons are not moderated or slowed down prior to interacting with fuel. They have large conversion ratios, and are suitable as breeders. To maintain the fast neutron (high energy) spectrum required, sodium, an excellent coolant with low elastic scattering cross section, is a desirable coolant. Also, sodium permits higher temperature operation, hence greater heat-to-electricity conversion efficiency.

Generally, fast reactors have small negative reactivity temperature coefficients. Their cores are small, with very high power density, perhaps as great as 4000 kW per gallon of coolant. Potential fuel exposures are expected to be great, possibly in excess of 50,000 MWd per ton.

Although the first nuclear generated electricity was by a breeder, the EBR-1, at the NRTS in Idaho in December, 1951, there are only two breeders in commercial operation in the world as of 1977 (one Russian, one French). A commercial American breeder, the Enrico Fermi Fast Breeder Reactor, was built near Detroit and began commercial operation in 1966. After a short operating lifetime, however,

the ^{238}U, might be wrapped around the outside of the fuel; this is known as a blanket. The fuel assemblies may contain 70 to 80% of fertile material. The objective is to obtain a high breeding gain or excess of new fuel over that consumed; in other words the breeding ratio minus 1.0 is the breeding gain. Breeding ratios of > 1.25 are usually considered economic; the Fermi reactor had a breeding gain of 1.2.

An important concept with respect to breeding is the doubling time - the time required to create an excess of fissile material equal to that in the core at the beginning of life (BOL). The doubling time, D, is a measure of the overall plant efficiency and can be expressed mathematically as shown in Equation 12-8-3:

$$D = \frac{M}{GP(1 + \alpha)} \qquad \text{Eq. 12-8-3}$$

where

D = doubling time, days.
M = mass of material required, grams.
G = breeding gain.
P = power level, megawatts.
α = ratio of fission to capture in fuel.

EXAMPLE
Given: A reactor with an initial charge of 2000 kg of ^{239}Pu; M then, is 2,000,000 g; breeding ra-

it suffered a blockage in flow (a plate that had been installed in the core for safety purposes fell into the coolant flow channel), which resulted in a partial meltdown of a few fuel elements and sufficient damage to cause the reactor to be peramenenty shut down.

A primary function of a breeder reactor is production of fuel - that is, fissile material - by neutron capture in fertile or fuel producing materials. Two fertile materials are known and plentiful: ^{238}U and ^{232}Th. These nuclides produce fissile materials according to the following reactions:

$$^{238}_{92}U + ^{1}_{0}n \rightarrow ^{239}_{92}U \xrightarrow[23.5\ min]{\beta^-} ^{239}_{93}Np \xrightarrow[2.3\ days]{\beta^-} ^{239}_{94}Pu$$

(24,000 years)

$$^{232}_{90}Th + ^{1}_{0}n \rightarrow ^{233}_{90}Th \xrightarrow[23.5\ min]{\beta^-} ^{233}_{91}Pa \xrightarrow[27.4\ days]{\beta^-} ^{233}_{93}U.$$

Neutron capture occurs with fast neutrons, hence breeders are fast reactors. Obviously, for a breeder to be useful - that is, produce more fuel than it consumes - the conversion ratio must be > 1. Another unit, however, the breeding ratio, is used with breeder reactors; the breeding ratio is simply the number of fissile atoms produced relative to the number of the same kind that have been consumed. Thus, the term breeder, in its strictest sense, applies only to reactors in which the fuel produced is the same as than being consumed.

In a breeder design, the fertile material, for example

tio is 1.3, G = 0.3; power level is 1000 MWth; $\alpha = 0.086$.

Find: Doubling time, D, in days:

Solution:

$$D = \frac{2{,}000{,}000}{0.3 \times 1000 \times (1 + 0.086)}$$

$$= 6139 \text{ days}$$

EXERCISES

1. Contrast important operating characteristics of LWR's and GCR's.
2. A geothermal steam well produces dry steam at 250°C, which can be used to power a turbine; the steam is cooled to 98°C; find theoretical upper efficiency of a geothermal plant producing electricity with this steam.
3. A reactor operates at 650°F and achieves a ΔT of 280°; find Carnot efficiency:
4. Determine doubling time for a breeder that has the following characteristics:

   Breeding gain = 0.75.

   Fission-to-capture ratio = 0.063.

   Initial loading = 870 kg of ^{239}Pu.

   Power level = 248 MWth.

   If breeding ratio is 1.25, what happens to doubling time?

5. Use Equation 12-8-2; compute conversion ratio for a reactor using 40% enriched uranium and natural uranium. (Assume a fuel-moderator density of 0.9.)

MATERIALS

None.

PROCEDURES

None.

DATA TABLE

None.

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Yevick, John G. "Breeder Reactors." In Human and Ecologi- Effects of Nuclear Power Plants, Chapter 9. Edited by L.A. Sagan. Springfield, IL: Charles C. Thomas, 1974.

1. On the basis of their design features, list differences between a gas-cooled and light-water cooled reactor:

2. List basic operating parameters of PWR's, BWR's and GCR's:

3. Calculate Carnot efficiency of a typical steam-electric system; (assume typical nuclear plant temperatures):

4. Briefly explain meaning of conversion ratio for operating reactors (as defined in this module):

5. List three advantages of heavy-water moderation:
   (1)

   (2)

   (3)

6. Briefly explain how a fast reactor works:

7. Give basic equation(s) for breeding:

8. Briefly describe (in terms used in this module) effects of voids on BWR, PWR and GCR operation:

NUCLEAR TECHNOLOGY

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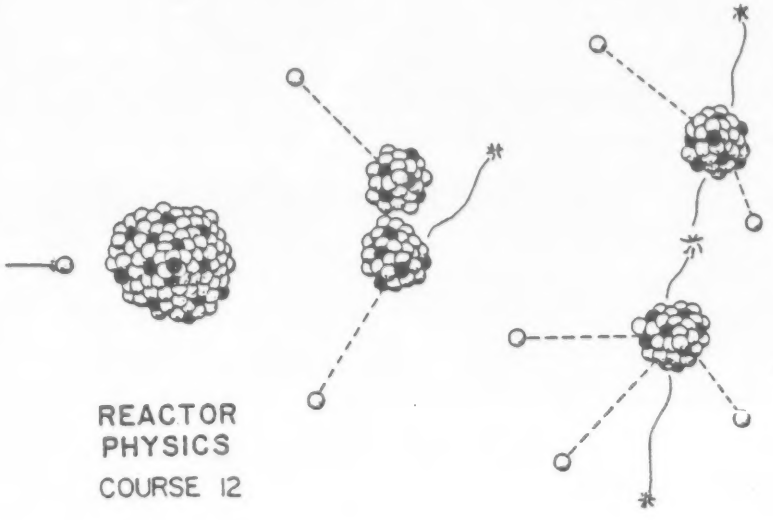
COURSE 12 - REACTOR PHYSICS



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NUCLEAR TECHNOLOGY SERIES

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for

COURSE 12, REACTOR PHYSICS (EIGHT MODULES)

NOTE: Each module in this Course is numbered separately, and the page numbers begin with page 1 in each module. Therefore, this Index lists for each entry the module number (underlined), followed by a comma and the page number. Figures and tables are listed by their numbers, followed by the module and page number in parentheses. A Master Index is available in a separate volume. This general index merges all index entries from the 35 Nuclear Courses. It is a valuable research tool, with which the student or researcher can locate all the references to any subject within the 35 Courses.

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